Answer on Question #85286 - Programming & Computer Science - Algorithms

Question 85286:

Prove the following using mathematical induction:

1. $(ab)^n = a^n b^n$ for every natural number n
2. $1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$
3. $1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$

Answer:

1. $(ab)^1 - a^1 b^1 = 0$

$(ab)^{n+1} - a^{n+1} b^{n+1} = ab(ab)^n - aa^n bb^n = ab((ab)^n - a^n b^n) = ab * 0 = 0$

2. $1^3 - 1^2 = 0$

$1^3 + 2^3 + 3^3 + \cdots + (n+1)^3 - \big(1 + 2 + 3 + \cdots + (n+1)\big)^2 = \big((1^3 + 2^3 + 3^3 + \cdots + n^3) + 1 + n^3 + 3 * n^2 + 3n\big) - \big((1 + 2 + 3 + \cdots + n)^2 - (n+1)^2 - 2(n+1)(1 + 2 + 3 + \cdots + n)\big) = 1 + n^3 + 3 * n^2 + 3n - (n+1)^2 - 2(n+1)(1 + 2 + 3 + \cdots + n) = 1 + n^3 + 3 * n^2 + 3n - n^2 - 2n - 1 - 2(n+1)n(n+1)/2 = n^3 + 2 * n^2 + n - (n+1)^2 n = 0$

3. $(2 * 1 - 1) - 1^2 = 0$

$1 + 3 + 5 + 7 + \cdots + (2(n+1) - 1) - (n+1)^2 = 1 + 3 + 5 + 7 + \cdots + (2n - 1) + (2n + 2 - 1) - n^2 - 2n - 1 = (2n + 2 - 1) - 2n - 1 = 0$