Answer on Question #48633, Physics, Computational Physics

A helium filled balloon (mass = 0.2 kg) is rising in the air with the buoyant force = Fb = [1.29e^(-1.21h)]g where h is in km.

How would I go about finding an expression for the velocity as a function of height by integrating acceleration?

Solution:

$$F_b = [1.29e^{-1.21h}]g$$

The equation of motion

$$ma = F_b - mg$$

Thus,

$$a = \frac{F_b}{m} - g = \frac{[1.29e^{-1.21h}]g}{m} - g = g\left(\frac{1.29}{m}e^{-1.21h} - 1\right)$$

$$a = g(6.45e^{-1.21h} - 1)$$

$$a(h) = \frac{dv}{dt} = \frac{dv}{dh}\frac{dh}{dt} = v\frac{dv}{dh}$$

Thus, integrating

$$\int_0^h a(h)dh = \int_0^v vdv$$

$$g\int_0^h (6.45e^{-1.21h} - 1)dh = \int_0^v vdv$$

$$g(6.45e^{-1.21h} - 1)|_0^h = \frac{v^2}{2}$$

$$g(-h - 5.33058\, e^{-1.21\, h} + 5.33058) = \frac{v^2}{2}$$

$$v = \sqrt{2g(5.33 - h - 5.33e^{-1.21\, h})}$$