Question #85012— Physics — Quantum Mechanics

For a motion of a particle of mass μ in a spherically symmetric potential show that L^2 and L_z commute with the Hamiltonian.

Solution

Hamiltonian of the particle in spherically symmetric potential $V(r)$ have form in spherical coordinates

$$H = -\frac{\hbar^2}{2\mu}\Delta + V(r) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{1}{2\mu r^2}\hat{L}^2 + V(r),$$

where $\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$.

and L_z in spherical coordinates $L_z = -i\hbar\frac{\partial}{\partial\phi}$.

Functions L_z, $\hat{L}^2$ commute with any function of r

$$[\hat{L}^2, f(r)] = [L_z, f(r)] = 0,$$

So

$$[H, \hat{L}^2] = [\frac{1}{2\mu r^2}\hat{L}^2, \hat{L}^2] = 0,$$

$$[H, L_z] = [\frac{1}{2\mu r^2}\hat{L}^2, L_z] = 0,$$

because $\hat{L}^2 \neq \hat{L}^2(\phi)$

Answer: $[H, L_z] = [H, \hat{L}^2] = 0$

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