Answer on Question #85011 Physics / Quantum Mechanics

Calculate the mean kinetic and potential energies of a simple harmonic oscillator which is in its ground state.

Solution:

In the ground state the wave function and energy of a simple harmonic oscillator

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

$$E_0 = \frac{\hbar\omega}{2}$$

The mean kinetic energy

$$\langle K \rangle = \int_{-\infty}^{\infty} \psi_0(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_0(x)dx$$

$$= \frac{\hbar^2}{2m}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \exp\left(-\frac{m\omega x^2}{\hbar}\right)\left(\frac{m\omega}{\hbar} - \frac{m^2\omega^2}{\hbar^2}x^2\right)dx$$

$$= \frac{\hbar^2}{2m}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}}\left(\frac{m\omega}{\hbar}\sqrt{\frac{\pi\hbar}{m\omega}} - \frac{m^2\omega^2}{\hbar^2}\frac{\hbar}{2m\omega}\sqrt{\frac{\pi\hbar}{m\omega}}\right) = \frac{\hbar\omega}{4}$$

The mean potential energy

$$\langle V \rangle = E_0 - \langle K \rangle = \frac{\hbar\omega}{2} - \frac{\hbar\omega}{4} = \frac{\hbar\omega}{4}$$

Answer: $\langle K \rangle = \langle V \rangle = \frac{\hbar\omega}{4}$