Answer on Question #84690 Physics / Quantum Mechanics

The wave front for a particle is defined by:

$$\Psi(x) = \begin{cases} N\cos\left(\frac{2\pi x}{L}\right), & \text{for } -L/4 \le x \le L/4 \\ 0, & \text{otherwise} \end{cases}$$

Determine:

i) the normalization constant N

ii) the probability that the particle will be found between $x = 0$ and $x = L/8$.

Solution:

i) The normalization condition (total probability=1)

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1$$

We have

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = N^2 \int_{-\frac{L}{4}}^{\frac{L}{4}} \cos^2\left(\frac{2\pi x}{L}\right) dx$$

$$= \frac{N^2}{2} \int_{-\frac{L}{4}}^{\frac{L}{4}} \left(1 + \cos\left(\frac{4\pi x}{L}\right)\right) dx = \frac{N^2 L}{4} = 1$$

Thus, the normalization constant

$$N = \sqrt{\frac{4}{L}}$$

ii) The probability that the particle will be found between $x = 0$ and $x = L/8$

$$P = \int_0^{\frac{L}{8}} |\Psi(x)|^2 dx = \frac{4}{L}\int_0^{\frac{L}{8}} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{2}{L}\int_0^{\frac{L}{8}} \left(1 + \cos\left(\frac{4\pi x}{L}\right)\right) dx$$

$$= \frac{2}{L}\left(\frac{L}{8} + \frac{L}{4\pi}\sin\left(\frac{4\pi x}{L}\right)\Big|_0^{\frac{L}{8}}\right) = \frac{2}{L}\left(\frac{L}{8} + \frac{L}{4\pi}\sin\left(\frac{\pi}{2}\right)\right) = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\pi}\right) = 0.41$$

Answer: $N = \sqrt{\frac{4}{L}}, \quad P = 0.41$