

### Answer on Question #84512 Physics / Atomic and Nuclear Physics

The wave front for a particle is defined by:

$$\psi(x) = \begin{cases} N \cos\left(\frac{2\pi x}{L}\right), & \text{for } -L/4 \leq x \leq L/4 \\ 0, & \text{otherwise} \end{cases}$$

Determine:

- i) the normalization constant  $N$
- ii) the probability that the particle will be found between  $x = 0$  and  $x = L/8$ .

**Solution:**

- i) The normalization condition

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

So

$$\begin{aligned} 1 &= N^2 \int_{-\frac{L}{4}}^{\frac{L}{4}} \cos^2\left(\frac{2\pi x}{L}\right) dx \\ &= \frac{N^2}{2} \int_{-\frac{L}{4}}^{\frac{L}{4}} \left(1 + \cos\left(\frac{4\pi x}{L}\right)\right) dx = \frac{N^2 L}{4} \end{aligned}$$

Thus, the normalization constant

$$N = \sqrt{\frac{4}{L}}$$

- ii) The probability that the particle will be found between  $x = 0$  and  $x = L/8$

$$\begin{aligned} P &= \int_0^{\frac{L}{8}} |\psi(x)|^2 dx = \frac{4}{L} \int_0^{\frac{L}{8}} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{2}{L} \int_0^{\frac{L}{8}} \left(1 + \cos\left(\frac{4\pi x}{L}\right)\right) dx \\ &= \frac{2}{L} \left( \frac{L}{8} + \frac{L}{4\pi} \sin\left(\frac{4\pi x}{L}\right) \Big|_0^{\frac{L}{8}} \right) = \frac{2}{L} \left( \frac{L}{8} + \frac{L}{4\pi} \sin\left(\frac{\pi}{2}\right) \right) = \frac{1}{2} \left( \frac{1}{2} + \frac{1}{\pi} \right) = 0.41 \end{aligned}$$

**Answer:**  $N = \sqrt{\frac{4}{L}}$ ,  $P = 0.41$