Answer on Question #84014 – Physics – Classical Mechanics

The particle with mass m is affected by the force (the force law is $F(r) = -k/r^2$), starting at rest at a distance d from the center of the force. Find the time it takes until the particle falls onto the center of the force. (Question source: A.P. Arya, *Introduction to Classical Mechanics*)

Solution:

By integrating the force, we can find the potential energy $V(r)$:

$$V(r) = -\int F(r)\,\mathrm{d}r = \int \frac{k}{r^2}\,\mathrm{d}r = -\frac{k}{r} \quad ,$$

where we have chosen the integration constant so that the potential energy is zero at infinity. Since the particle starts at rest and since the force is central, the particle moves radially. From the conservation of energy, we have

$$E = \frac{1}{2} m\dot{r}^2 + V(r) = \frac{1}{2} m\dot{r}^2 - \frac{k}{r} = \mathrm{const} \quad .$$

To find the constant of energy E in this equation, we note that the velocity $\dot{r}$ is zero at the distance d, hence, $E = -k/d$. This enables us to find the velocity as a function of distance:

$$\dot{r} = -\sqrt{\frac{2k}{m}\left(\frac{1}{r} - \frac{1}{d}\right)} \quad .$$

To find the time t it takes for a particle to move from $r = d$ to $r = 0$, we simply integrate:

$$t = \int_d^0 \frac{\mathrm{d}r}{\dot{r}} = -\int_0^d \frac{\mathrm{d}r}{\dot{r}} = \int_0^d \frac{\mathrm{d}r}{\sqrt{\frac{2k}{m}\left(\frac{1}{r} - \frac{1}{d}\right)}} = \sqrt{\frac{md}{2k}} \int_0^d \frac{\mathrm{d}r}{\sqrt{\frac{d}{r} - 1}} = \sqrt{\frac{md}{2k}}\, d \int_0^1 \frac{\mathrm{d}x}{\sqrt{\frac{1}{x} - 1}} = \frac{\pi d}{2}\sqrt{\frac{md}{2k}} \quad ,$$

where we have used the integral $\int_0^1 \mathrm{d}x/\sqrt{1/x - 1} = \pi/2$.

Answer: $\frac{\pi d}{2}\sqrt{\frac{md}{2k}}$.