Answer on Question # 83382, Physics / Mechanics | Relativity

Question 1. *A $m = 1\,kg$ object slides to the right on a surface with kinetic friction coefficient of $\mu = 0.25$. The initial speed of the object when it makes contact with a light spring (k spring $= 50\,N/m$) is $v_0 = 3\,m/s$. The object comes to rest after the spring has been compressed a distance d. The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's un-stretched position. Finally, the object comes to rest a distance D to the left of the un-stretched spring. Find d, D, and the speed v at the un-stretched position when the object is moving to the left.*

Solution. The kinetic energy of the object was spent on friction and compressing the spring. Friction did the work, μmgd and the spring (elastic force) did the work $kd^2/2$. Hence,

$$mv_0^2/2 = \mu mgd + kd^2/2 \Rightarrow d^2 + 2\mu mgd/k - mv_0^2 = 0 \Rightarrow d = \frac{1}{2}\left(-\mu mg/k \pm \sqrt{(\mu mg/k)^2 + mv_0^2/k}\right),$$

$$d > 0 \Rightarrow d = -0.25 \cdot 9.8/50 \pm \sqrt{(0.25 \cdot 9.8/50)^2 + 3^2/50} \approx 0.378\,m.$$

In the moment, when the spring at the un-stretched position, the spring did work which was spent on friction and the kinetic energy of the object. Then,

$$kd^2/2 = \mu mgd + mv^2/2 \Rightarrow v^2 = \left(kd^2 + 2\mu mgd\right)/m,$$

$$v > 0 \Rightarrow v = \sqrt{\left(kd^2 - 2\mu mgd\right)/m} = \sqrt{(50 \cdot 0.378^2 - 2 \cdot 0.25 \cdot 9.8 \cdot 0.378)} \approx 2.3\,ms^{-1}.$$

When the object stopped, the kinetic energy was spent on friction. So,

$$mv^2/2 = \mu mgD \Rightarrow D = mv^2/(2\mu mg) = 2.3^2/(2 \cdot 0.25 \cdot 9.8) \approx 1.08\,m.$$

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