

Question 1. *Proved that in AC circuit average AC power consumed is zero.*

Solution. A circuit element dissipates or produces power according to $P = IV$, where I is the current through the element and V is the voltage across it. Let $p(t) = i(t)v(t)$.

$P_{ave} = \frac{1}{T} \int_0^T p(t) dt$, where $T = \frac{2\pi}{\omega}$ is the period of the oscillations. With the substitutions $v(t) = V_0 \sin \omega t$ and $i(t) = I_0 \sin(\omega t - \phi)$, this integral becomes

$$P_{ave} = \frac{I_0 V_0}{T} \int_0^T \sin(\omega t - \phi) \sin \omega t dt,$$

$$P_{ave} = \frac{1}{2} I_0 V_0 \cos \phi.$$

For the resistor $\phi = 0$ and $P_{ave} = \frac{1}{2} I_0 V_0$.

For a capacitor $\phi = \frac{\pi}{2}$ and for an inductor $\phi = -\frac{\pi}{2}$, so $P_{ave} = 0$ for both.

The phase angle for an AC generator may have any value. If $\cos \phi > 0$, the generator produces power; if $\cos \phi < 0$, it absorbs power. \square