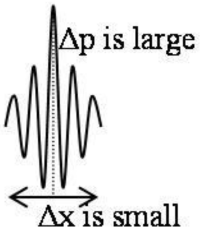
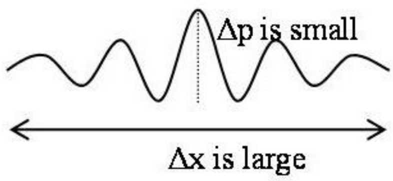
Question #82597, Physics / Quantum Mechanics |

Prove Heisenberg's uncertainty relation using the concept of wavepacket.

Solution



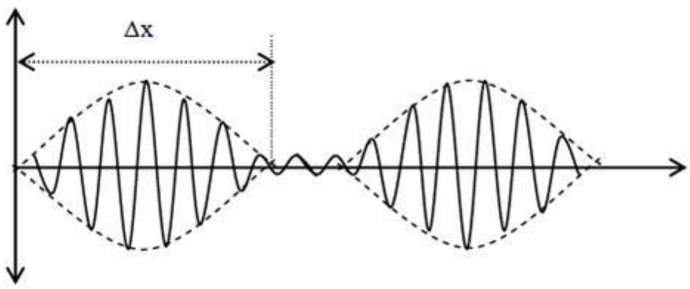


Pic. 1 **Pic. 2**

In quantum mechanics a particle is described by a wave packet, which represents and symbolizes all about particle and moves with group velocity.

Pic. 1 – Narrow wave packet and **Pic. 2** – wide wave packet

For a large wave packet with many crests the velocity spread is very small so that the particle velocity can be fairly determined, but the position of the particle became completely uncertain. On the other hand if we consider infinitely small wave packet the position of the particle become certain but the velocity became quite uncertain.



The position of the particle can be located anywhere in the wave packet, along the x-axis the length of the wave packet is measured between two nodes (where amplitude becomes almost zero)

The amplitude of the wave $2A\cos\left[\frac{\Delta\omega}{2}t-\frac{\Delta k}{2}x\right]$

As 2A will never be zero, $\cos\left[\frac{\Delta\omega}{2}t-\frac{\Delta k}{2}x\right]=0$

Or

$$\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right] = \frac{\pi}{2};\ \frac{3\pi}{2};\ \frac{5\pi}{2};\ \frac{7\pi}{2}, \dots \dots \dots .\frac{(2n+1)\pi}{2}$$

Where n = 1, 2, 3, 4…..

$$\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_1\right] = \frac{\pi}{2}\ (1)$$

$$\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_2\right] = \frac{3\pi}{2}\ (2)$$

Subtracting above equations (1) and (2), we get

$$(x_1 - x_2)\frac{\Delta k}{2} = \pi \rightarrow \Delta x \Delta k = 2\pi \ \rightarrow\ \Delta x \frac{2\pi}{\Delta\lambda} = 2\pi \ \rightarrow \Delta x = \Delta\lambda$$

$$\Delta x = \frac{\hbar}{\Delta p} = \Delta x \Delta p = \hbar$$

Or

$$\boldsymbol{\Delta x \Delta p} \geq \hbar$$

Answer provided by https://www.AssignmentExpert.com