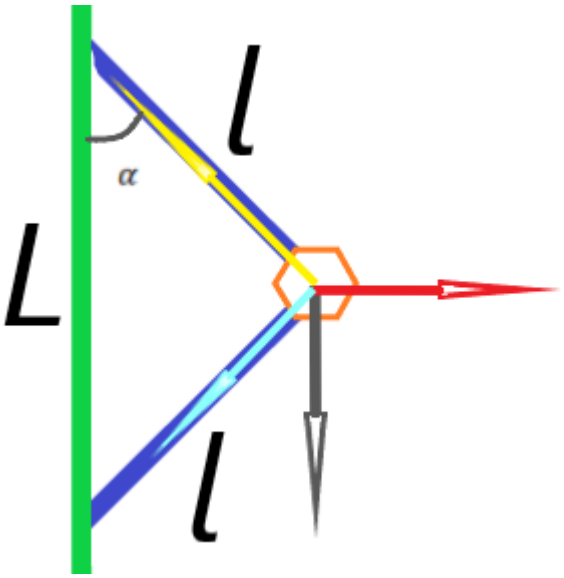
Answer on Question #82368 - Physics - Mechanics – Relativity

A 4.00-kg object is attached to a vertical rod by two strings, as in Figure P6.11. The object rotates in a horizontal circle at constant speed 6.00 m/s. Find the tension in (a) the upper string and (b) the lower string

Solution



There are 4 forces acting on the body: tensions T_1, T_2 acting upwards and downwards correspondingly (yellow and blue), force of gravity mg and the centripetal force $m\ (v^2/r)$. Write them according to Newton's second law by axes:

$$Ox:\ -T_1 \sin\alpha - T_2 \sin\alpha + m\frac{v^2}{r} = 0,$$

$$Oy:\quad T_1 \cos\alpha - T_2 \cos\alpha - mg = 0.$$

Solve the system:

$$T_1 = \frac{mg}{\cos\alpha}\left(1 - \frac{\sin\alpha}{2}\right) + \frac{mv^2}{2r\sin\alpha},$$

$$T_2 = \frac{m}{2\sin\alpha}\left(\frac{v^2}{r} - g\tan\alpha\right).$$

Suppose that we know the distance L between the two points where the string is tied, and the distance l between these points and the body, hence

$$\sin\alpha = \frac{\sqrt{4l^2 - L^2}}{2l}, \qquad \cos\alpha = \frac{L}{2l}, \qquad \tan\alpha = \frac{\sqrt{4l^2 - L^2}}{L}, \qquad r = \frac{\sqrt{4l^2 - L^2}}{2}.$$

Just substitute α or l or L.

Answer

$$T_1 = \frac{39.2}{\cos\alpha}\left(1 - \frac{\sin\alpha}{2}\right) + \frac{72}{r\sin\alpha}, \qquad T_2 = \frac{2}{\sin\alpha}\left(\frac{36}{r} - 9.8\tan\alpha\right).$$