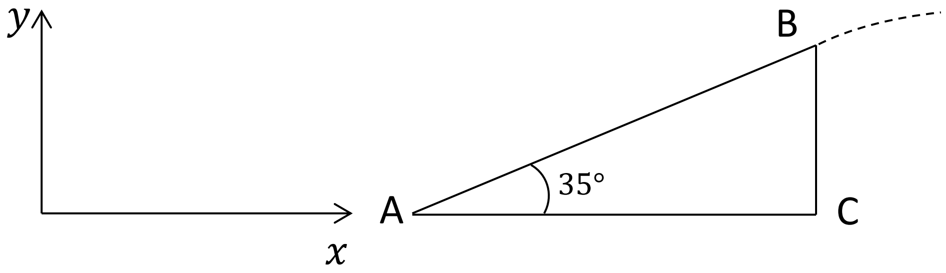
Answer on Question#82201 - Physics – Mechanics | Relativity

A test rocket is launched by accelerating it along a 200.0-m incline at 1.25 m/s^2 starting from rest at point A. The incline rises at 35˚ above the horizontal, and at the instant the rocket leaves it, its engines turn off and it is subject only to gravity (air resistance can be ignored). Find (a) the maximum height above the ground that the rocket reaches, and (b) the greatest horizontal range of the rocket beyond point A.

Solution:



Since $AB = 200$ m, thus we obtain

$$BC = AB\sin 35° = 114.7\text{ m}$$

$$AC = AB\cos 35° = 163.8\text{ m}$$

The a speed v_f, an initial speed v_i, a traveled path l and an acceleration a are related by the following expression

$$v_f^2 - v_i^2 = 2al$$

Since $v_i = 0\text{ m/s}$, $l = 200\text{ m}$ and $a = 1.25\text{ m/s}^2$, we obtain the speed at point B:

$$v_f = \sqrt{2al + v_i^2} = \sqrt{2\cdot 1.25\frac{\text{m}}{\text{s}^2}\cdot 200\text{ m} + \left(0\frac{\text{m}}{\text{s}}\right)^2} = 22.36\ \frac{\text{m}}{\text{s}}$$

The coordinates of the rocket after it passes point B is described by following system of equations (origin is at point A):

$$\begin{cases} y(t) = BC + v_f\sin 35°\, t - \dfrac{gt^2}{2}, \\ x(t) = AC + v_f\cos 35°\, t \end{cases}$$

where $g = 9.8\text{ m/s}^2$ – is the acceleration due to gravity and t – is time.

The vertical component of velocity is given by

$$v_y = v_f\sin 35° - gt$$

The maximum height is reached when $v_y = 0$, thus we obtain

$$t = \frac{v_f\sin 35°}{g} = \frac{22.36\ \frac{\text{m}}{\text{s}}\cdot\sin 35°}{9.8\ \frac{\text{m}}{\text{s}^2}} = 1.31\text{ s}$$

Substituting it into $y(t)$ we get

$$y_{max} = y(1.31\text{ s}) = 114.7\text{ m} + 22.36\ \frac{\text{m}}{\text{s}} \cdot \sin 35° \cdot 1.31\text{ s} - \frac{9.8\ \frac{\text{m}}{\text{s}^2} \cdot (1.31\text{ s})^2}{2}$$

$$y_{max} = 123.1\text{ m}$$

The rocket will fall when $y(t) = 0$, therefore

$$BC + v_f \sin 35°\, t - \frac{gt^2}{2} = 0$$

$$114.7\text{ m} + 12.8\ \frac{\text{m}}{\text{s}} \cdot t - \frac{9.8\ \frac{\text{m}}{\text{s}^2} \cdot t^2}{2} = 0$$

The positive root of this equation is $t = 6.3$ s. It is the time when the rocket falls on the ground after it passes point B. Substituting it into $x(t)$ we obtain the greatest horizontal range:

$$x_{max} = x(6.3\text{ s}) = 163.8\text{ m} + 18.3 \frac{\text{m}}{\text{s}} \cdot 6.3\text{ s} = 279.1\text{ m}$$

Answer: $y_{max} = 123.1$ m, $x_{max} = 279.1$ m.