Answer on Question #82132 - Physics - Atomic and Nuclear Physics

Question: *You want to find the half-life of an element. At 12 AM on the first day you find that the element has decayed 50000 times after 1 min. At 12 AM on the second day you find that the element has decayed 45000 times in 1 minute. What is the half-life of the element?*

Answer:

Solution of the problem is based on the utilization of the law of radioactive decay which states

$$N(t) = N_0 2^{-\frac{t}{T}}, \tag{1}$$

where N_0 is the number of the initially existing nuclei, $N(t)$ is the number of the not decayed nuclei by the time t, T is the half-life period.

Hence, the number of already decayed nuclei can be calculated as:

$$N_{dec}(t) = N_0 - N(t) = N_0(1 - 2^{-\frac{t}{T}}). \tag{2}$$

For the first observation we have:

$$t_1 = \Delta t = 1min, \quad \Delta N_1 = 50000, \tag{3}$$

$$\Delta N_1 = N_0(1 - 2^{-\frac{\Delta t}{T}}). \tag{4}$$

For the second observation we have:

$$t_2 = t_0 + \Delta t, \quad t_0 = 24h, \quad \Delta N_2 = 45000, \tag{5}$$

$$\Delta N_2 = N(t_0)(1 - 2^{-\frac{\Delta t}{T}}) = N_0 2^{-\frac{t}{T}}(1 - 2^{-\frac{\Delta t}{T}}), \tag{6}$$

where we utilize $N(t_0)$ as the number of still existing nuclei after 24 hours.

Dividing (4) by (6), we obtain:

$$\frac{\Delta N_1}{\Delta N_2} = \frac{N_0(1 - 2^{-\frac{\Delta t}{T}})}{N_0 2^{-\frac{t}{T}}(1 - 2^{-\frac{\Delta t}{T}})} = 2^{\frac{t}{T}}. \tag{7}$$

By solving (7) in respect to T, we obtain:

$$T = \frac{t_0}{\log_2 \frac{\Delta N_1}{\Delta N_2}} = \frac{24h}{\log_2 \frac{50000}{45000}} \approx 158h. \tag{8}$$

So, the half-life of the element is around 158 hours.