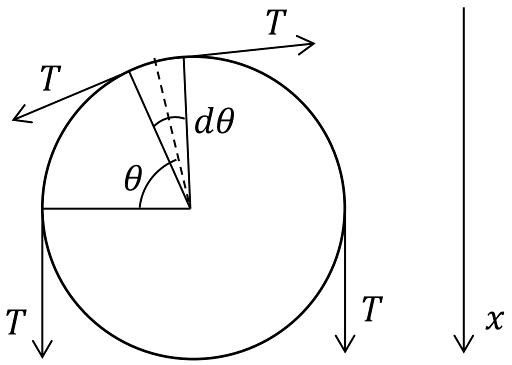
Answer on Question#82098 - Physics – Mechanics | Relativity

A flexible massless rope is placed over a cylinder of radius R. A tension T is applied to each end of the rope, which remains stationary (see the figure below). Show that each small segment dθ of the rope in contact with the cylinder pushes against the cylinder with a force T dθ in the radial direction. By integration of the forces exerted by all the small segments, show that the net vertical force on the cylinder is 2T and the net horizontal force is zero

Solution:



Let's consider a small angle $d\theta$ as shown in the picture above. The normal to the cylinder surface at the middle of the segment (dashed line) bisects the angle $d\theta$. Thus projections on the normal of two forces T and T that act on this segment are the same and given by

$$T_n = T\sin\frac{d\theta}{2}$$

Since we can choose angle $d\theta$ to be arbitrary small, we can use the following approximation

$$\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$$

Therefore the total projection of the mentioned forces on the normal is given by

$$dF_t = 2T_n = 2T\sin\frac{d\theta}{2} = 2T\frac{d\theta}{2} = Td\theta$$

The projection of this force on the x-axis is

$$dF_x = dF_t\sin\theta = T\sin\theta\, d\theta$$

According to the task the rope makes half-turn around the cylinder, thus the angle θ goes from 0 to π. Integrating dF_x in this range we obtain

$$F_x = \int_0^\pi dF_x = \int_0^\pi T\sin\theta\, d\theta = -T\cos\theta|_0^\pi = -T(-1-1) = 2T$$

Answer: $dF_t = Td\theta$, $F_x = 2T$.