Answer on Question 82036, Physics, Astronomy, Astrophysics

Question:

From home plate to the center field wall at a ball park is 130 meters. When a batter hits a long drive the ball leaves his bat 1 meter off the ground with a velocity of 40 meters per second at 28 degrees above the horizontal. The center field wall is 2.6 meters high. Does he hit a home run?

Solution:

Let's, consider the motion of the ball in two dimensions:

$$x = v_{0x}t = v_0 cos\theta t, \qquad (1)$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = y_0 + v_0 sin\theta t - \frac{1}{2}gt^2, (2)$$

here, $x = 130\,m$ is the horizontal distance traveled by the ball to the center field wall, $v_0 = 40\ m/s$ is the initial velocity of the ball, $\theta = 28^\circ$ is the launch angle, y is the height of the ball, $y_0 = 1.0\,m$ is the initial height of the ball above the ground, $g = 9.8\ m/s^2$ is the acceleration due to gravity and t is the time.

Let's find the height of the ball as a function of horizontal distance by eliminating the time. Let's express the time from the equation (1) and substitute it into the equation (2):

$$t = \frac{x}{v_0 cos\theta},$$

$$y = y_0 + v_0 sin\theta \cdot \frac{x}{v_0 cos\theta} - \frac{1}{2}g\left(\frac{x}{v_0 cos\theta}\right)^2,$$

$$y = y_0 + xtan\theta - \frac{1}{2}g\left(\frac{x}{V_0 cos\theta}\right)^2 =$$

$$= 1.0\,m + 130\,m \cdot tan28^\circ - \frac{1}{2}\cdot 9.8\,\frac{m}{s^2}\cdot\left(\frac{130\,m}{40\,\frac{m}{s}\cdot cos28^\circ}\right)^2 =$$

$$= 1.0\,m + 2.73\,m = 3.73\,m.$$

Since, the height of the ball is greater than the height of the center field wall, the ball will clear this wall and the batter will hit a home run.

Answer:

$y = 3.73\ m$, the batter will hit the home run.