Question #82016, Physics / Mechanics | Relativity

A train start from rest at station A and is uniformly accelerated until it reaches a speed of 108 km/h. It then travels at this speed until the breaks are applied and the train is then uniformly retarded until it stops at station B. The magnitude of this retardation is twice the magnitude of the initial acceleration. The distance between the stations is 12 km and the time taken for the journey is 10 minutes. Find the:

1. Time spent on each of the three stages of the journey.

2. Initial acceleration.

3. Final retardation

Solution

The total distance:

$$d = \frac{at_1^2}{2} + vt_2 + \frac{1}{2}(2a)t_3^2$$

$$t_1 = \frac{v}{a}, t_3 = \frac{v}{2a}, t_2 = t - t_1 - t_3 = t - \frac{v}{a} - \frac{v}{2a} = t - \frac{3v}{2a}.$$

$$d = \frac{a}{2}\left(\frac{v}{a}\right)^2 + v\left(t - \frac{3v}{2a}\right) + \frac{1}{2}(2a)\left(\frac{v}{2a}\right)^2 = \frac{3}{4}\frac{v^2}{a} + v\left(t - \frac{3v}{2a}\right)$$

$$d = vt - \frac{3}{4}\frac{v^2}{a}$$

$$a = \frac{3}{4}\frac{v^2}{vt - d} = \frac{3}{4}\frac{\left(\frac{108}{3.6}\right)^2}{\left(\frac{108}{3.6}\right)(600) - 12000} = 0.1125\frac{m}{s^2}.$$

1.

$$t_1 = \frac{v}{a} = \frac{\left(\frac{108}{3.6}\right)}{0.1125} = 267\ s = \frac{\left(\frac{108}{3.6}\right)}{0.1125}\frac{1}{60}\ min = 4.4\ min.$$

$$t_3 = \frac{v}{2a} = \frac{t_1}{2} = 2.2\ min.$$

$$t_2 = 12 - 4.4 - 2.2 = 5.4\ min.$$

2. Initial acceleration:

$$a = 0.1125\frac{m}{s^2}.$$

3. Final retardation:

$$2a = 2(0.1125) = 0.225\frac{m}{s^2}.$$