Answer on Question #81943 Physics / Other

Range of 300 km. Height of 100 km. Maximum initial velocity in m/s and angle.

Solution:

In the case of projectile motion the range R and maximum height H are given by expressions

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

So

$$\frac{R}{H} = \frac{\sin 2\theta}{\frac{\sin^2 \theta}{2}} = \frac{4 \sin \theta \cos \theta}{\sin^2 \theta} = 4 \cot \theta$$

$$\cot \theta = \frac{R}{4H} = \frac{300}{4 \times 100} = 0.75, \qquad \theta = \cot^{-1} 0.75 = 53.1°$$

Since

$$\sin^2 \theta = \frac{1}{\cot^2 \theta + 1} = \frac{1}{\left(\frac{R}{4H}\right)^2 + 1} = \frac{16H^2}{R^2 + 16H^2}$$

we get

$$H = \frac{v_0^2 \times \frac{16H^2}{R^2 + 16H^2}}{2g}$$

Finally

$$v_0 = \sqrt{gH\left(\frac{R^2}{8H^2} + 2\right)} = \sqrt{9.8 \times 100\,000 \times \left(\frac{300^2}{8 \times 100^2} + 2\right)} = 1750 \frac{\text{m}}{\text{s}}$$

Answers: $\mathbf{1750 \frac{m}{s}, 53.1°}$