Answer on Question #81908 - Physics - Mechanics – Relativity

A ring takes time t1 in slipping down an inclined plane of length L, whereas it takes time t2 in rolling down the same plane. The ratio of t1 and t2 is

Solution

In this problem we will use:

- r — radius of the ring;
- m — mass of the ring;
- I — moment of inertia of the ring;
- μ — coefficient of kinetic friction;
- α — angle of the slope;
- ε — angular acceleration while the ring is rolling;
- f — force of friction.

When the ring is *slipping* its time is

$$t_1 = \sqrt{\frac{2L}{a_1}},$$

where a_1 – its acceleration. Determine it using Newton's second law:

$$ma_1 = mg\sin\alpha - \mu mg\cos\alpha,$$

$$a_1 = g(\sin\alpha - \mu\cos\alpha),$$

$$t_1 = \sqrt{\frac{2L}{g(\sin\alpha - \mu\cos\alpha)}}.$$

Now calculate the time for the second situation:

$$t_2 = \sqrt{\frac{2L}{a_2}},$$

where a_2 – acceleration of the *rolling* ring. Determine it from the equality of torques provided <u>by the force of friction</u> and <u>by the moment of inertia</u>:

$$f \cdot r = I \cdot \varepsilon,$$

$$\varepsilon = \frac{fr}{I}.$$

On the other hand,

$$\varepsilon = \frac{a_2}{r} \Rightarrow \frac{fr}{I} = \frac{a_2}{r}.$$

According to Newton's second law in this case,

$$ma_2 = mg\sin\alpha - f.$$

Express f from the equation right above and obtain a_2:

$$a_2 = \frac{r^2 mg\sin\alpha}{I + mr^2}.$$

The moment of inertia for rings is

$$I = mr^2,$$

so

$$a_2 = \frac{g\sin\alpha}{2}$$

and

$$t_2 = \sqrt{\frac{4L}{g\sin\alpha}}.$$

Finally,

$$\frac{t_1}{t_2} = \sqrt{\frac{2L}{g(\sin\alpha - \mu\cos\alpha)} \cdot \frac{g\sin\alpha}{4L}} = \sqrt{\frac{\sin\alpha}{2(\sin\alpha - \mu\cos\alpha)}}.$$

If $\mu = 0$,

$$\frac{t_1}{t_2} = \sqrt{\frac{1}{2}}.$$

Answer

$$\frac{t_1}{t_2} = \sqrt{\frac{\sin\alpha}{2(\sin\alpha - \mu\cos\alpha)}}.$$