Question #81867, Physics / Optics

A gun barrel of mass 600 kg has a recoil spring of stiffness 294 kN/meter. If the barrel recoils 1.3m on firing determine (i) the initial recoil velocity of the barrel (ii) the critical damping co-efficient of the dash pot which is engaged at the end of the recoil strike (iii) the equation of motion of the gun barrel if the time of recoils is of time period.

Solution

(i) The natural frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{294000}{600}} = 22.14\frac{rad}{s}.$$

From conservation of energy, we have

$$\frac{mv^2}{2} = \frac{kx^2}{2}$$

$$v = \omega x = (22.14)(1.3) = 28.8\frac{m}{s}.$$

(ii) At critical damping, the damping factor $= 1$. The critical damping coefficient is therefore:

$$c = 2m\zeta\omega = 2(600)(1)(22.14) = 26500\frac{kg}{s}.$$

(iii) We set the time when the barrel reaches the position x=1.3 to be t=0. At critical damping, the motion of barrel could be expressed as:

$$x(t) = (c_1 + c_2 t)e^{-\omega t}$$

The initial conditions:

$$x(0) = c_1 = 1.3.$$

$$v(0) = v = c_2 = 28.8.$$

Thus,

$$x(t) = (1.3 + 28.8t)e^{-22.14t}$$