What is the mean mass density for a super massive black hole with total mass of $1*10^8$ mass of sun inside the Schwarzschild radius?

Solution:

Mass density formula looks like this: $\rho=\frac{M}{V}$, where M – mass of a black hole, V – volume of black hole. It's considered, that visible radius of black hole is equal to its Schwarzschild radius, so volume is equal to $V=\frac{4}{3}\pi {r_g}^3$.

r_g – Schwarzschild's radius, which is equal to $r_g=\frac{2GM}{c^2}$, where G – gravitational constant, c – speed of light, M – mass of the black hole. Substituting M and V in starting formula we get:

$$\rho=\frac{M}{V}=\frac{10^8*M_\odot}{\frac{4}{3}\pi {r_g}^3}=\frac{3*10^8*M_\odot}{4\pi\left(\frac{2GM}{c^2}\right)^3}=\frac{3*10^8*M_\odot}{4\pi\frac{8G^3M^3}{c^6}}=\frac{3*10^8*M_\odot*c^6}{32\pi G^3M^3}=$$

$$\frac{3*10^8*M_\odot*c^6}{32\pi G^3(10^8*M_\odot)^3}=\frac{3*c^6}{32\pi G^3(10^8*M_\odot)^2}=\frac{3*c^6}{32\pi G^3*10^{16}*M_\odot^2};$$

(Where $M_\odot$ is mass of Sun).

Substituting numerical values, we get:

$$\rho=\frac{3*c^6}{32\pi G^3*10^{16}*M_\odot^2}=\frac{3*(3*10^8 m/s)^6}{32*3.141592*\left(6.67*10^{-11}\frac{m^3}{s^2*kg}\right)^3*10^{16}*3,9204*10^{60}kg^2}=$$

$$\frac{3*729*10^{48}\frac{m^6}{s^6}}{32*3.141592*2.967*10^{-31}\frac{m^9}{s^6*kg^3}*10^{16}*3,9204*10^{60}kg^2}=\frac{2.187*10^{51}\frac{m^6}{s^6}}{2.983*10^{-13}\frac{m^9}{s^6*kg^3}*3,9204*10^{60}kg^2}=$$

$$\frac{2.187*10^{51}\frac{m^6}{s^6}}{2.983*10^{-13}\frac{m^9}{s^6*kg^3}*3,9204*10^{60}kg^2}=\frac{2.187*10^{51}\frac{m^6}{s^6}}{1.17*10^{48}\frac{m^9}{s^6*kg}}=1869.23\frac{kg}{m^3}$$

Answer:

The mean mass density for black hole, mentioned in task, is $1869.23\frac{kg}{m^3}$.