Obtain expressions in component form for the position vectors having the following polar coordinates.
(a) $12.6 \mathrm{~m}, 140^{\circ}$ counterclockwise from the $+x$ axis
(b) $4.00 \mathrm{~cm}, 50.0^{\circ}$ counterclockwise from the $+x$ axis
(c) 20.0 in., $220^{\circ}$ counterclockwise from the $+x$ axis

## Solution.

We denote the length of the vector $\vec{R}$ by $R$ and the angle between the positive direction of $x$ axis and the vector $\vec{R}$ by $\theta$. Let $\overrightarrow{\mathrm{l}}$ and $\overrightarrow{\mathrm{j}}$ be the unit vectors directed along the $x$-axis and $y$-axis respectively.

The projections of the vector $\overrightarrow{\mathrm{R}}$ on the axis:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{x}}=\mathrm{R} \cos \theta \\
& \mathrm{R}_{\mathrm{y}}=\mathrm{R} \sin \theta
\end{aligned}
$$

Vector in the component notation:

$$
\begin{gathered}
\vec{R}=\left(R_{x}, R_{y}\right)=\vec{\imath} R_{x}+\vec{\jmath} R_{y} \\
\vec{R}=(R \cos \theta, R \sin \theta)=\vec{\imath} R \cos \theta+\vec{\jmath} R \sin \theta
\end{gathered}
$$

(a).
$\mathrm{R}=12.6 \mathrm{~m} ; \theta=140^{\circ}$

$$
\begin{aligned}
& R \cos \theta=12.6 \times \cos 140^{\circ} \approx-9.7 m \\
& \mathrm{R} \sin \theta=12.6 \times \sin 140^{\circ} \approx 8.1 \mathrm{~m} \\
& \vec{R}=(-9.7 \mathrm{~m}, 8.1 \mathrm{~m})=-\overrightarrow{\mathrm{l}} \times(9.7 \mathrm{~m})+\vec{\jmath} \times(8.1 \mathrm{~m})
\end{aligned}
$$

Answer: $\overrightarrow{\mathrm{R}}=(-9.7 \mathrm{~m}, 8.1 \mathrm{~m})=-\overrightarrow{\mathrm{i}} \times(9.7 \mathrm{~m})+\overrightarrow{\mathrm{j}} \times(8.1 \mathrm{~m})$
(b).
$R=4.00 \mathrm{~cm} ; \theta=50^{\circ}$

$$
\begin{gathered}
\text { R }=4.0 \mathrm{~cm} \underbrace{\prime \prime} \\
\mathrm{R} \cos \theta=4.00 \times \cos 50^{\circ} \approx-2.57 \mathrm{~cm} \\
\mathrm{R} \sin \theta=4.00 \times \sin 50^{\circ} \approx 3.06 \mathrm{~cm} \\
\overrightarrow{\mathrm{R}}=(-2.57 \mathrm{~cm}, 3.06 \mathrm{~cm})=-\overrightarrow{\mathrm{i}} \times(2.57 \mathrm{~cm})+\overrightarrow{\mathrm{\jmath}} \times(3.06 \mathrm{~cm})
\end{gathered}
$$

Answer: $\overrightarrow{\mathrm{R}}=(-2.57 \mathrm{~cm}, 3.06 \mathrm{~cm})=-\overrightarrow{\mathrm{\imath}} \times(2.57 \mathrm{~cm})+\vec{\jmath} \times(3.06 \mathrm{~cm})$
(c).
$R=20.0 \mathrm{in} ; \theta=220^{\circ}$

$R \cos \theta=20.0 \times \cos 220^{\circ} \approx-15.3$ in
$\mathrm{R} \sin \theta=20.0 \times \sin 220^{\circ} \approx-12.9 \mathrm{in}$

$$
\overrightarrow{\mathrm{R}}=(-15.3 \mathrm{in}, \quad-12.9 \mathrm{in})=-\overrightarrow{\mathrm{\imath}} \times(15.3 \mathrm{in})-\overrightarrow{\mathrm{\jmath}} \times(12.9 \mathrm{in})
$$

Answer: $\overrightarrow{\mathrm{R}}=(-15.3 \mathrm{in}, \quad-12.9 \mathrm{in})=-\overrightarrow{\mathrm{l}} \times(15.3 \mathrm{in})-\overrightarrow{\mathrm{j}} \times(12.9 \mathrm{in})$
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