Answer on Question #80438, Physics / Electromagnetism

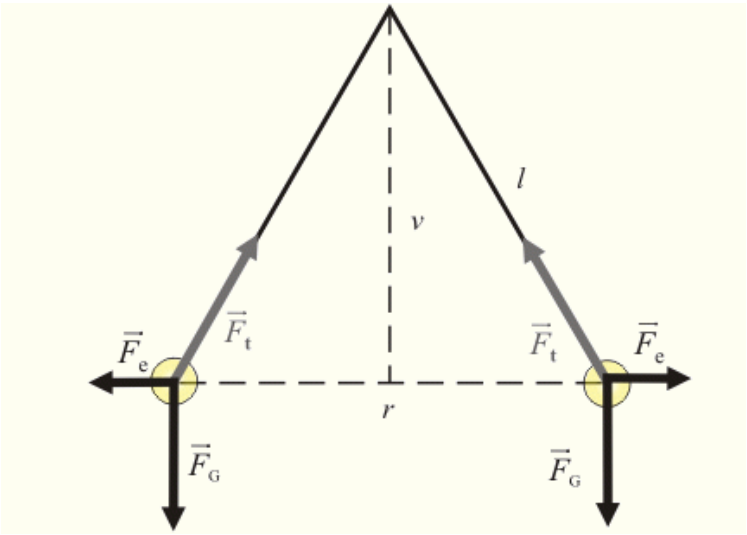
Two metal spheres of identical mass m = 3.20 g are suspended by light strings 0.500 m in length. The left-hand sphere carries a charge of 0.785 μC, and the right-hand sphere carries a charge of 1.51 μC.

What is the equilibrium separation between the centers of the two spheres?

Solution:

Two forces act on each ball hanging on the string: a force of gravity and tension of the string. The balls are also charged, so they repel one another with electric force. We determine its size using Coulomb's law.

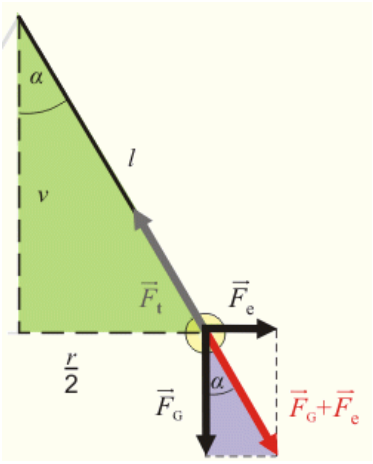
Both balls are at rest, so the net force must be zero, thus, the vector sum of electric force and the force of gravity must be of the same size and opposite direction as the tension of the string.



The size of the repulsive electric forces is determined from Coulomb's law:

$$F_e = k\frac{q_1 q_2}{r^2}$$

From triangles



$$\tan\alpha = \frac{F_e}{F_G}$$

and

$$\tan\alpha = \frac{r}{2v} = \frac{r}{2\sqrt{l^2 - (r/2)^2}} = \frac{r}{\sqrt{4l^2 - r^2}}$$

So,

$$\frac{F_e}{F_G} = \frac{r}{\sqrt{4l^2 - r^2}}$$

$$k\frac{q_1 q_2}{r^2 mg} = \frac{r}{\sqrt{4l^2 - r^2}}$$

It's complicated equation.

We can assume that the angle between the strings is small. For small angles

$$\tan\alpha = \sin\alpha = \frac{r}{2}\cdot\frac{1}{l} = \frac{r}{2l}$$

So,

$$\frac{r}{2l} = k\frac{q_1 q_2}{r^2 mg}$$

and

$$r = \sqrt[3]{k\frac{2lq_1 q_2}{mg}} = \sqrt[3]{9\times10^9 \times \frac{2\times0.500\times0.785\times10^{-6}\times1.51\times10^{-6}}{3.2\times10^{-3}\times9.8}} = 0.698\,m$$

Answer: $0.698\,m$