Answer on Question #79706 - Physics - Electric Circuits

Question: *The magnitude of work done in taking a unit positive charge in electric field E from point A to point B is given by:*
$W = -\int_A^B \vec{E} \cdot d\vec{r}$
Show that the value of the line integral of the electric field (right hand side of the above equation) does not depend on the path taken to move the unit positive charge from point A to B.

Answer:

The electrostatic field $\vec{E}$ can be determined by the gradient of the potential as [1]

$$\vec{E} = -\nabla\varphi. \tag{1}$$

As a result, the initial line integral transforms to the form as follows (here we put $q = 1$ as it is stated in the question above):

$$W = \int_A^B \nabla\varphi \cdot d\vec{r}. \tag{2}$$

According to the gradient theorem [2], the line integral (2) can be calculated as:

$$W = \int_A^B \nabla\varphi \cdot d\vec{r} = \varphi(B) - \varphi(A). \tag{3}$$

Consequently, the work done by moving a point unit charge in the electrostatic field $\vec{E}$ does not depend on the path. It depends on the initial and final positions only.

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[1] (Electronic resource) https://en.wikipedia.org/wiki/Electric_potential
[2] (Electronic resource) https://en.wikipedia.org/wiki/Gradient_theorem