Two guitar wires A and B identical in all respects except that the diameter of A is twice the diameter of B and are subjected to same tension. The ratio, fundamental frequency produced by A / fundamental frequency produced by B.

Solution:

The fundamental frequency

$$f = \frac{v}{2L}$$

Here $v = \sqrt{\frac{T}{\frac{m}{L}}}$ is the wave velocity.

Finally

$$f = \frac{\sqrt{\frac{T}{\frac{m}{L}}}}{2L}$$

T is a string tension

m is a string mass

L is a string length

Thus

$$\frac{f_A}{f_B} = \frac{\frac{\sqrt{\frac{T_A}{\frac{m_A}{L_A}}}}{2L_A}}{\frac{\sqrt{\frac{T_B}{\frac{m_B}{L_B}}}}{2L_B}} = \sqrt{\frac{m_B}{m_A}}$$

Since

$$m = \text{density} \times \text{volume} = \rho V = \rho L A = \rho L \frac{\pi d^2}{4}$$

So

$$\frac{f_A}{f_B} = \sqrt{\frac{d_B^2}{d_A^2}} = \frac{d_B}{d_A} = \frac{d_B}{2d_B} = \frac{1}{2}$$

Answer: $\frac{f_A}{f_B} = \frac{1}{2}$