Answer on Question #79566, Physics / Electromagnetism

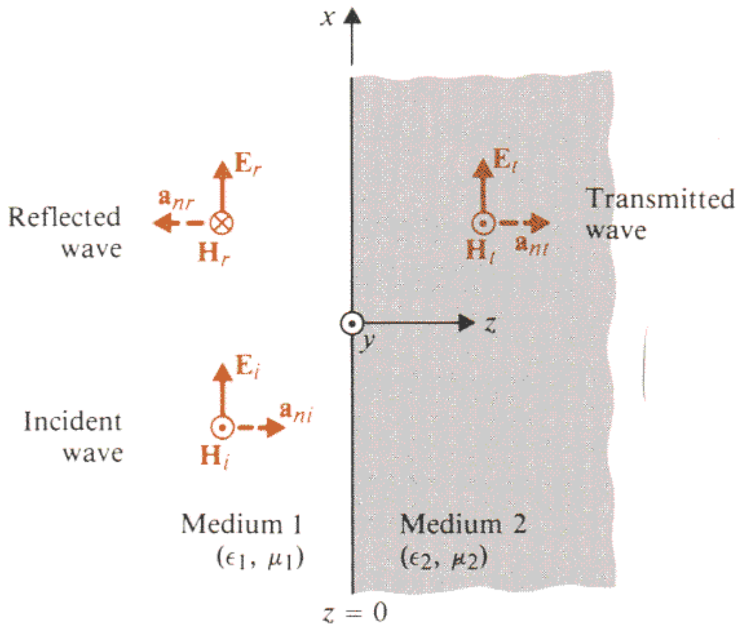
A uniform plane wave of 100 kHz travelling in free space strikes a large block of a material having $\varepsilon = 4\varepsilon_0$, $\mu = 9\mu_0$ and $\sigma = 0$ normal to the surface. If the incident magnetic field vector is given by

$$\vec{\mathbf{B}} = 10^{-6}\cos(\omega t - \beta y)\,\hat{\mathbf{z}}\text{ tesla}$$

write the complete expressions for the incident, reflected, and transmitted field vectors.

Solution:

When an electromagnetic wave travelling in one dielectric medium impinges on another dielectric medium with a different intrinsic impedance, part of the incident wave is reflected and part is transmitted.



The incident wave fields:

$$\vec{\mathbf{E}}_i = E_0 e^{-\gamma_1 z}\cdot\hat{\mathbf{x}}$$

$$\vec{\mathbf{H}}_i = \frac{E_0}{\eta_1} e^{-\gamma_1 z}\cdot\hat{\mathbf{y}}$$

The reflected wave fields:

$$\vec{\mathbf{E}}_t = \Gamma E_0 e^{-\gamma_1 z}\cdot\hat{\mathbf{x}}$$

$$\vec{\mathbf{H}}_t = -\Gamma\frac{E_0}{\eta_1} e^{\gamma_1 z}\cdot\hat{\mathbf{y}}$$

The transmitted wave fields:

$$\vec{\mathbf{E}}_t = \tau E_0 e^{-\gamma_2 z}\cdot\hat{\mathbf{x}}$$

$$\vec{\mathbf{H}}_t = \tau\frac{E_0}{\eta_2} e^{-\gamma_2 z}\cdot\hat{\mathbf{y}}$$

where Γ is the reflection coefficient, τ – transmission coefficient, $\eta = \sqrt{\mu/\varepsilon}$.

(i) The incident wave fields:

$\vec{\mathbf{E}}_i$ has the magnitude

$$E_0 = H_0\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{B_0}{\sqrt{\mu_0\varepsilon_0}} = \frac{10^{-6}}{\sqrt{\mu_0\varepsilon_0}} = 10^{-6}\times 3\times 10^{8} = 3\times 10^{2}$$

$$\vec{\mathbf{E}}_i = E_0 e^{-\gamma_1 z}\cdot\hat{\mathbf{x}} = 3\times 10^{2}\cos(\omega t-\beta y)\,(-\hat{\mathbf{x}})$$

$$\beta = \frac{\omega}{c} = \frac{2\pi\times 100\times 10^{3}}{3\times 10^{8}} = \frac{\pi}{1500}$$

So,

$$\vec{\mathbf{E}}_i = 3\times 10^{2}\cos\left(2\pi\times 10^{5}t-\frac{\pi}{1500}y\right)(-\hat{\mathbf{x}})\ \text{V/m}$$

$$\vec{\mathbf{H}}_i = \frac{\vec{\mathbf{B}}_i}{\mu_0} = \frac{10^{-6}}{\mu_0}\cos(\omega t-\beta y)\,\hat{\mathbf{z}} = \frac{10^{-6}}{4\pi\times 10^{-7}}\cos(\omega t-\beta y) =$$

$$= \frac{2.5}{\pi}\cos\left(2\pi\times 10^{5}t-\frac{\pi}{1500}y\right)\hat{\mathbf{z}}\ \text{A/m}$$

(ii) We assume that the incident electric field is reflected with a reflection coefficient Γ and transmitted with a transmitted with a transmission coefficient τ . That implies that if the electric field intensity of the incident, reflected and transmitted waves at the boundary (z = 0) are E_{i0} , E_{r0} and E_{t0} respectively, then $E_{r0} = \Gamma E_{i0}$ and $E_{t0} = \tau E_{i0}$.

$$\tau = 1 + \Gamma$$

and

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} =$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{9\mu_0}{4\varepsilon_0}} = \frac{3}{2}\times 120\pi = 180\pi\ \Omega$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{180-120}{180+120} = 0.2$$

$$\tau = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{2*180\pi}{300\pi} = 1.2$$

Using the general properties above, we conclude that ($\beta_r = -\beta$ because wave propagates in opposite direction)

$$\vec{\mathbf{E}}_r = 0.6\times 10^{2}\cos\left(2\pi\times 10^{5}t+\frac{\pi}{1500}y\right)(\hat{\mathbf{x}})\ \text{V/m}$$

$$\vec{\mathbf{H}}_r = \frac{1.5}{\pi}\cos\left(2\pi\times 10^{5}t+\frac{\pi}{1500}y\right)\hat{\mathbf{z}}\ \text{A/m}$$

(iii)

$$\beta_2 = 2\pi f\sqrt{\mu_2\varepsilon_2} = 2\pi f\sqrt{36\mu_0\varepsilon_0} = 2*\pi*100*10^{3}*6*\frac{1}{3*10^{8}} = \frac{\pi}{250}$$

So,

$$\vec{\mathbf{E}}_t = 3.6\times 10^{2}\cos\left(2\pi\times 10^{5}t-\frac{\pi}{250}y\right)(-\hat{\mathbf{x}})\ \text{V/m}$$

$$\vec{\mathbf{H}}_t = \frac{3}{\pi}\cos\left(2\pi \times 10^5 t - \frac{\pi}{250}y\right)\hat{\mathbf{z}}\ \mathrm{A/m}$$

Answer:

Incident wave:

$$\vec{\mathbf{E}}_i = 3 \times 10^2 \cos\left(2\pi \times 10^5 t - \frac{\pi}{1500}y\right)(-\hat{\mathbf{x}})\ \ \mathrm{V/m}$$

$$\vec{\mathbf{H}}_i = \frac{2.5}{\pi}\cos\left(2\pi \times 10^5 t - \frac{\pi}{1500}y\right)\hat{\mathbf{z}}\ \ \mathrm{A/m}$$

Reflected wave:

$$\vec{\mathbf{E}}_r = 0.6 \times 10^2 \cos\left(2\pi \times 10^5 t + \frac{\pi}{1500}y\right)(\hat{\mathbf{x}})\ \ \mathrm{V/m}$$

$$\vec{\mathbf{H}}_r = \frac{1.5}{\pi}\cos\left(2\pi \times 10^5 t + \frac{\pi}{1500}y\right)\hat{\mathbf{z}}\ \mathrm{A/m}$$

Transmitted wave:

$$\vec{\mathbf{E}}_t = 3.6 \times 10^2 \cos\left(2\pi \times 10^5 t - \frac{\pi}{250}y\right)(-\hat{\mathbf{x}})\ \ \mathrm{V/m}$$

$$\vec{\mathbf{H}}_t = \frac{3}{\pi}\cos\left(2\pi \times 10^5 t - \frac{\pi}{250}y\right)\hat{\mathbf{z}}\ \mathrm{A/m}$$