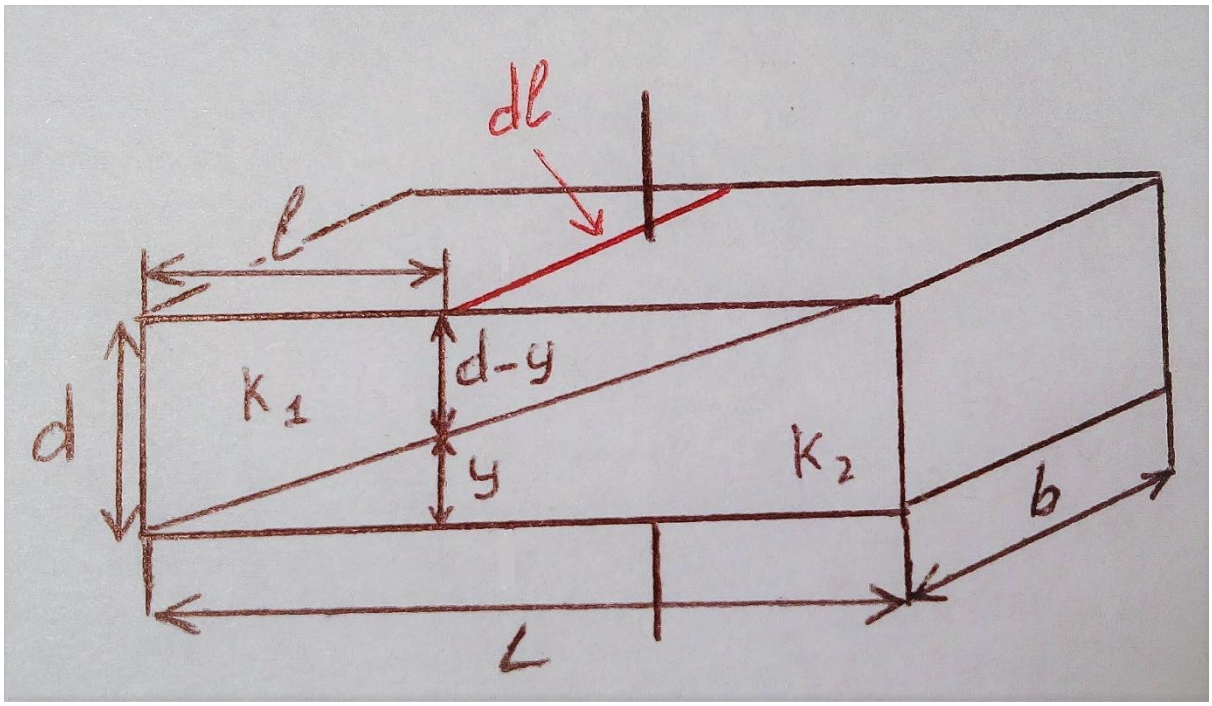
Answer on Question #79496, Physics / Electric Circuits

Find capacitance between the parallel plates if the dielectric constant is varying from k1 to K2 diagonally

Solution

Agree that length of both plates is L, breadth is b, distance between plates is d. Their area then is $A = b \cdot L$. Imagine that the capacitor is split on $\mathrm{d}l$ long pieces alongside (in parallel) into equal pairs of two capacitors joined in series: the first small capacitor with K_1 dielectric constant on thickness $d - y$ and the second small capacitor with K_2 dielectric of thickness y.



Hence, Plates' area of these smaller capacitors is $b \cdot \mathrm{d}l$. Aggregate capacity of these two capacitors is

$$\frac{1}{\mathrm{d}C} = \frac{1}{\varepsilon_0 b \cdot \mathrm{d}l}\left(\frac{d-y}{K_1} + \frac{y}{K_2}\right),$$

$$y = d\frac{l}{L}.$$

Then

$$\mathrm{d}C = \frac{\varepsilon_0 A K_1 K_2 \cdot \mathrm{d}l}{d} \cdot \frac{K_1 K_2}{K_2 L + l(K_1 - K_2)},$$

Then integrate this from 0 to L by $\mathrm{d}l$:

$$\int \mathrm{d}C = \int_0^L \frac{\varepsilon_0 A K_1 K_2}{d} \cdot \frac{K_1 K_2}{K_2 L + l(K_1 - K_2)}\,\mathrm{d}l,$$

$$C = \frac{\varepsilon_0 A K_1 K_2}{d(K_1 - K_2)} \cdot ln\frac{K_1}{K_2},$$

Answer

$$C = \frac{\varepsilon_0 A K_1 K_2}{d(K_1 - K_2)} \cdot ln \frac{K_1}{K_2}$$