Answer on Question #79364 - Physics - Molecular Physics / Thermodynamics

Question: *Two litres of hydrogen at a pressure of 10^5 Pa expands adiabatically to 1.5 times the initial volume. Find the work done?*

Answer:

The first law of thermodynamics [1] can be formulated in the following way:

$$Q = \Delta U + A, \quad (1)$$

where Q is amount of heat supplied to the system, ΔU is the change in the internal energy of the system, A is the work done by the system. For adiabatic process $Q = 0$, so that the work can be done due to the loss of the internal energy only:

$$A = -\Delta U\ . \quad (2)$$

Hydrogen is a diatomic (ideal) gas, and each of its molecules possesses $i = 5$ degrees of freedom. Hence, the change in its internal energy should be written as:

$$\Delta U = \frac{5}{2}\nu R(T_2 - T_1)\,. \quad (3)$$

According to Mendeleev-Clapeyron equation

$$p_1V_1 = \nu RT_1\,,$$

$$p_2V_2 = \nu RT_2\,, \quad (4)$$

Hence,

$$\nu R(T_2 - T_1) = p_2V_2 - p_1V_1\,. \quad (5)$$

There is also one more useful relation for the adiabatic processes:

$$pV^{\gamma} = Const\,, \quad (6)$$

where γ is the adiabatic index (= heat capacity ratio). For diatomic ideal gases $\gamma = \frac{7}{5}$ (see [2] for details).

As a result, one can show that:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma}. \quad (7)$$

Combining (2)-(3), (5), (7) together and taking into account the relation $V_2 = 1.5V_1 = \frac{3}{2}V_1$, we deduce

$$A = -\frac{5}{2}(p_2V_2 - p_1V_1) = \frac{5}{2}p_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right] = \frac{5}{2}p_1V_1\left[1-\left(\frac{2}{3}\right)^{2/5}\right]. \quad (8)$$

Substituting the numerical values, we obtain:

$$A \approx \frac{5}{2}\cdot 10^5\cdot 2\cdot 10^{-3}(1-0.85) = 425\ J. \quad (9)$$

[1] (Electronic resource) https://en.wikipedia.org/wiki/First_law_of_thermodynamics

[2] (Electronic resource) https://en.wikipedia.org/wiki/Heat_capacity_ratio