Answer on Question #79316- Physics- Quantum Mechanics

Question: *1. If sigma x, sigma y, and sigma z are three components of a pauli spin matrix sigma, then show that [sigma x, sigma y]=2i sigma z; [sigma y, sigma z]= 2i sigma x*

Answer:

In the so-called σ_z–representation the Pauli matrices can be written in the form as follows [1]:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1}$$

One should also recall the definition of the commutator in quantum mechanics:

$$[\mathrm{A, B}] = \mathrm{AB\text{-}BA}\,, \tag{2}$$

where A and B are two arbitrary quantum operators.

Substituting the matrices (1) into the definition (2), we obtain:

$$[\sigma_x, \sigma_y] = \sigma_x\sigma_y - \sigma_y\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = \\ = 2i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\sigma_z \tag{3}$$

In a similar fashion we get:

$$[\sigma_y, \sigma_z] = \sigma_y\sigma_z - \sigma_z\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2i \\ 2i & 0 \end{pmatrix} = \\ = 2i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 2i\sigma_x \tag{4}$$

[1] (Electronic resource) https://en.wikipedia.org/wiki/Pauli_matrices