Answer on Question#79283 - Physics - Electromagnetism

A stiff metal rod kept over two knife edges of length $L = 1\ \text{m}$. Rod carries current of $I = 16\ \text{A}$ and rolls over rails without slipping due to uniform magnetic field of $B = 0.5\ \text{T}$ perpendicular pointing downwards. Rod starts from rest and attains speed of $K/\sqrt{5}$ when leaves rails. Find value of K.

Solution:

Let's denote the length of the rod as l, then the work done magnetic field is given by

$$A_m = IBlL$$

The translational kinetic energy:

$$E_t = \frac{1}{2}mv^2,$$

where m – mass of the rod, v – linear velocity of the rod.

The rotational kinetic energy:

$$E_r = \frac{1}{2}I\omega^2,$$

where I – moment of inertia of the rod around its axis, ω – angular velocity of the rod.

The moment of inertia of a cylinder of mass m and radius r is given by

$$I = \frac{1}{2}mr^2$$

Since the rod doesn't slip, its linear and angular velocities are related as follows

$$v = \omega r$$

Total kinetic energy:

$$E_k = E_t + E_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2$$

According to the law of conservation of energy the kinetic energy gained by the rod is equal to the work done by the magnetic field:

$$E_k = A_m$$

Thus

$$\frac{3}{4}mv^2 = IBlL$$

$$v = \sqrt{\frac{4IBlL}{3m}} = \sqrt{\frac{20IBlL}{3m}}/\sqrt{5}$$

Therefore

$$K = \sqrt{\frac{20IBlL}{3m}}$$

For numerical answer the length of the rod l and its mass are needed.

Answer: $K = \sqrt{\frac{20IBlL}{3m}}$.