Answer on Question#78945 - Physics - Astronomy - Astrophysics

If a satellite collects the rate of unforeseen intergenerative dust ((dM)/(dt))=av in the free interiors, then the satellite will accelerate.

Solution:

Dust creates the net force F given by:

$$F = -\frac{dM}{dt}v$$

According to the Newton's second law we have

$$M\frac{dv}{dt} = F = -\frac{dM}{dt}v \qquad (1)$$

$$M\frac{dv}{dt} + \frac{dM}{dt}v = 0$$

$$\frac{d(Mv)}{dt} = 0$$

$$Mv = p_0$$

p_0 – initial momentum of the satellite (before encountering dust).

Since $\frac{dM}{dt} = av$, we obtain

$$M\frac{1}{a}\frac{dM}{dt} = p_0$$

$$\frac{d(M^2)}{dt} = 2ap_0$$

$$M = \sqrt{M_0^2 + 2ap_0 t},$$

where M_0 – initial mass of the satellite.

Substituting this and $\frac{dM}{dt} = av$ in to equation (1) we obtain:

$$\sqrt{M_0^2 + 2ap_0 t}\frac{dv}{dt} = -av^2$$

$$-\frac{dv}{v^2} = a\frac{dt}{\sqrt{M_0^2 + 2ap_0 t}}$$

$$\frac{1}{v} = \frac{1}{p_0}\sqrt{M_0^2 + 2ap_0 t}$$

$$v = \frac{1}{\frac{\sqrt{M_0^2 + 2ap_0 t}}{p_0}} = \frac{v_0}{\sqrt{1 + \frac{2ap_0 t}{M_0^2}}}$$

Thus the acceleration:

$$\frac{dv}{dt} = -\frac{\frac{ap_0v_0}{M_0^2}}{\left(1+\frac{2ap_0t}{M_0^2}\right)^{3/2}}$$

Answer: acceleration: $-\frac{\frac{ap_0v_0}{M_0^2}}{\left(1+\frac{2ap_0t}{M_0^2}\right)^{3/2}}$.