Answer on Question#78859 - Physics - Molecular physics - Thermodynamics

A centrifugal compressor which represents an open system takes in air at a pressure of $P_i = 2$ bar and a temperature of 28˚C at the rate of 1.4 m/s. Compression takes place according to the law $PV^{1.3} = C$ and the delivery pressure is $P_f = 3.5$ bar. Determine

(i) the input power and

(ii) the heat transfer rate.

Solution:

Initial temperature in Kelvin:

$$T_i = 28℃ = 301\text{ K}$$

The elementary work done by compressor is given by

$$dA = PdV,$$

Since $P = C/V^{1.3}$, we obtain

$$A = C\int_{V_i}^{V_f} \frac{dV}{V^{1.3}} = \frac{10}{3}C\left(V_i^{-0.3} - V_f^{-0.3}\right) = \frac{10}{3}CV_i^{-0.3}\left(1 - \left(\frac{V_i}{V_f}\right)^{0.3}\right)$$

According to the given law of compression:

$$\left(\frac{V_f}{V_i}\right)^{1.3} = \frac{P_i}{P_f}$$

Using this expression we can rewrite the work in the following way:

$$A = P_iV_i\left(1 - \left(\frac{P_f}{P_i}\right)^{\frac{3}{13}}\right)$$

The change of the internal energy of the ν moles of air is given by

$$\Delta U = 2.5\nu R\Delta T = 2.5\nu R\left(T_f - T_i\right)$$

Using the ideal gas law ($PV = \nu RT$) we obtain

$$\nu = \frac{P_iV_i}{RT_i}$$

Also due to $PV^{1.3} = C$ we have

$$T_f = T_i\left(\frac{P_f}{P_i}\right)^{\frac{23}{13}}$$

The heat transfer is given by

$$Q = A + \Delta U$$

The work for one cubic meter of air:

$$A_1 = 2\text{ bar} \cdot 1\text{ m}^3 \left(1 - \left(\frac{3.5\text{ bar}}{2\text{ bar}}\right)^{\frac{3}{13}}\right) = -13785\text{ J}$$

Change of internal energy of one cubic meter of air:

$$\Delta U_1 = 2.5 P_i V_i \left(\left(\frac{P_f}{P_i}\right)^{\frac{23}{13}} - 1\right) = 2.5 \cdot 2\text{ bar} \cdot 1\text{ m}^3 \left(\left(\frac{3.5\text{ bar}}{2\text{ bar}}\right)^{\frac{23}{13}} - 1\right) = 845.74\text{ kJ}$$

Thus (for one cubic meter)

$$Q_1 = A_1 + \Delta U_1 = -13785\text{ J} + 845.74\text{ kJ} = 832\text{ kJ}$$

The input power (since the rate of pump is 1.4 m^3/s):

$$N = -A_1 \cdot 1.4 \frac{\text{m}^3}{\text{s}} = 19.3\text{ kW}$$

The heat transfer rate (due to the cooling of compressed air):

$$\frac{dQ}{dt} = Q_1 \cdot 1.4 \frac{\text{m}^3}{\text{s}} = 1.165\text{ MW}$$

Answer: input power: 19.3 kW, heat transfer rate: 1.165 MW.