Answer on Question#78814 - Physics - Classical Mechanics

Given that for a damped oscillation $m = 250\text{g}$, $k = 85\ \text{N/m}$ $b = 75\ \text{g/s}$. Calculate the period of oscillation, the time taken for the amplitude of the damped oscillation to drop to half of its initial value, time taken for the mechanical energy to drop to half of its initial value.

Solution:

The equation of motion:

$$m\ddot{x} + b\dot{x} + kx = 0$$

Damping coefficient:

$$\gamma = \frac{b}{2m} = \frac{75\,\frac{\text{g}}{\text{s}}}{2 \cdot 250\ \text{g}} = 0.15\ \text{s}^{-1}$$

Undamped angular frequency:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{85\ \text{W/m}}{0.25\ \text{kg}}} = 18.4\ \text{s}^{-1}$$

The angular frequency of the damped oscillator is given by

$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2} = \sqrt{340\text{s}^{-2} - 0.0225\text{s}^{-2}} \approx \omega_0 = 18.4\ \text{s}^{-1}$$

Thus the period of oscillation

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{18.4\ \text{s}^{-1}} = 0.34\ \text{s}$$

Dependence of amplitude on time:

$$A(t) = A_0 e^{-\gamma t},$$

where A_0 – initial amplitude.

For $A(t) = \frac{1}{2}A_0$ we obtain

$$\frac{1}{2} = e^{-\gamma t_1}$$

$$t_1 = \frac{\ln 2}{\gamma} = \frac{\ln 2}{0.15\ \text{s}^{-1}} = 4.62\ \text{s}$$

Since the damping coefficient is small, the total energy for each oscillation can be calculated as the doubled potential energy of the oscillator $E = 2U = 2\frac{kx^2}{2} = kx^2$. Since x is proportional to A, we obtain

$$E = E_0 e^{-2\gamma t_2},$$

where E_0 – initial energy of the oscillator. Thus for $E = \frac{1}{2}E_0$ we get

$$\frac{1}{2} = e^{-2\gamma t_2}$$

$$t_2 = \frac{\ln 2}{2\gamma} = \frac{\ln 2}{2 \cdot 0.15\ \mathrm{s}^{-1}} = 2.3\ \mathrm{s}$$

Answer: $T = 0.34\ \mathrm{s}$, $t_1 = 4.62\ \mathrm{s}$, $t_2 = 2.3\ \mathrm{s}$.