Answer on Question#78812 - Physics - Classical Mechanics

The equation for a damper harmonic oscillator is given as $x = 10e^{-0.125t}\cos\left(\frac{\pi}{2}t\right)$. Calculate angular frequency, natural angular frequency, initial energy per unit mass of the damped oscillator, the damped time, the nature of the oscillation, the quality factor and the particles velocity at time equal zero.

Solution:

Since $\frac{\pi}{2}t = \omega_1 t$, the angular frequency is $\omega_1 = \frac{\pi}{2} = 1.57\ \mathrm{s}^{-1}$. Since $e^{-0.125t} = e^{-\gamma t}$ the damping coefficient is $\gamma = 0.125\ \mathrm{s}^{-1}$. The natural frequency ω_0 is given by

$$\omega_0 = \sqrt{\omega_1^2 + \gamma^2} = \sqrt{(1.57\ \mathrm{s}^{-1})^2 + (0.125\ \mathrm{s}^{-1})^2} = 1.57\ \mathrm{s}^{-1}$$

$t = 0$ s:

Position:

$$x(0) = 10\ \mathrm{m}$$

Velocity:

$$\dot{x}(0) = 10\left(-0.125e^{-0.125t}\cos\left(\frac{\pi}{2}t\right) - \frac{\pi}{2}e^{-0.125t}\sin\left(\frac{\pi}{2}t\right)\right)\Big|_{t=0} = -1.25\,\frac{\mathrm{m}}{\mathrm{s}}$$

Thus initial energy per unit mass is given by

$$\epsilon(0) = \frac{kx^2(0)}{2m} + \frac{\dot{x}^2(0)}{2} = \frac{x^2(0)}{2}\frac{k}{m} + \frac{\dot{x}^2(0)}{2}$$

Since $\omega_0 = \sqrt{k/m}$, we obtain

$$\epsilon(0) = \frac{x^2(0)\omega_0^2}{2} + \frac{\dot{x}^2(0)}{2} = \frac{(10\ \mathrm{m})^2(1.57\ \mathrm{s}^{-1})^2}{2} + \frac{\left(-1.25\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} = 124\,\frac{\mathrm{J}}{\mathrm{kg}}$$

The damped time

$$\tau = \frac{1}{\gamma} = \frac{1}{0.125\ \mathrm{s}^{-1}} = 8\ \mathrm{s}$$

Since $\omega_0 > \gamma$, the oscillator is underdamped.

The quality factor is given by

$$Q = \frac{\omega_0}{2\gamma} = \frac{1.57\ \mathrm{s}^{-1}}{2\cdot 0.125\ \mathrm{s}^{-1}} = 6.28$$

Answer: $\omega_1 = 1.57\ \mathrm{s}^{-1}$, $\omega_0 = 1.57\ \mathrm{s}^{-1}$, $\epsilon(0) = 124\ \mathrm{J/kg}$, $\tau = 8$ s, underdamped oscillator, $Q = 6.28$, $\dot{x}(0) = -1.25\,\frac{\mathrm{m}}{\mathrm{s}}$.