Answer on Question#78799 - Physics - Molecular Physics

Evaluate and compare the work and heat transfer when $m = 0.8$ kg of air, at a pressure of $P = 2.6$ Bar and a temperature of C degree, expand in a closed thermodynamic system to three times its initial volume: (1) according to Boyles law and, (2) according to Charles law.

The characteristic gas constant for air is $R = 200\ \mathrm{J/kg \cdot K}$ and its specific heat capacity at constant volume is $C_V = 700\ \mathrm{J/kg \cdot K}$.

Solution:

$T_i = 23\ \mathrm{°C} = 300\ \mathrm{K}$ – initial temperature.

(1) According to Boyles law $PV = const$. Thus the internal energy of the gas doesn't change and the heat transfer is given by the work of the gas, which in this case is

$$\Delta Q_B = mRT \ln \frac{V_f}{V_i},$$

where V_i – initial volume of the gas, V_f – final volume.

Since $V_f = 3V_i$, we obtain

$$\Delta Q_B = 0.8\ \mathrm{kg} \cdot 200 \frac{\mathrm{J}}{\mathrm{kg \cdot K}} \cdot 300\ \mathrm{K} \ln 3 = 52.7\ \mathrm{kJ}$$

(2) According to Charles law $V/T = const$, thus

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Since $V_f = 3V_i$, $T_f = 3T_i$. The heat transfer is given by

$$\Delta Q_C = P\Delta V + mC_V\Delta T$$

According to the ideal gas law $PV = mRT$, thus the first member (the work of the gas) in the upper equation can be rewritten in the following way:

$$A_C = P\Delta V = mR\Delta T = 0.8\ \mathrm{kg} \cdot 200 \frac{\mathrm{J}}{\mathrm{kg \cdot K}} \cdot 2 \cdot 300\ \mathrm{K} = 96\ \mathrm{kJ}$$

Therefore we obtain

$$\Delta Q_C = mR\Delta T + mC_V\Delta T = m(R + C_V)\Delta T = m(R + C_V)2T_i =$$
$$= 0.8\ \mathrm{kg}\left(200 \frac{\mathrm{J}}{\mathrm{kg \cdot K}} + 700 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}\right) 2 \cdot 300\ \mathrm{K} = 432\ \mathrm{kJ}$$

Thus the work of the gas is $A_C/\Delta Q_B = 96\ \mathrm{kJ}/52.7\ \mathrm{kJ} = 1.8$ times greater according to Charles law, than to Boyles law. Also the heat transfer in case of Charles law is 8.2 times greater than in case of Boyles law.

Answer:

(1) $\Delta Q_B = 52.7\ \mathrm{kJ}$
$A_B = \Delta Q_B = 52.7\ \mathrm{kJ}$

(2) $\Delta Q_C = 432\ \mathrm{kJ}$
$A_C = 96\ \mathrm{kJ}$