Answer on Question 76597, Physics, Molecular Physics, Thermodynamics

Question:

Titan, a satellite of Saturn, has a mean orbital radius of $1.22 \cdot 10^9\ m$. The orbital period of Titan is 15.95 days. Hyperion, another satellite of Saturn, orbits at a mean radius of $1.48 \cdot 10^9\ m$. Estimate the orbital period of Hyperion.

Solution:

The third Kepler's law of planetary motion states that the square of the orbital period of a planet is proportional to the cube of the semi-major axis (mean distance) of its orbit:

$$\frac{P_T^2}{a_T^3} = \frac{P_H^2}{a_H^3},$$

here, P_T is the orbital period of the Titan, P_H is the orbital period of the Hyperion, a_T is the mean distance of the Titan from the Saturn, a_H is the mean distance of the Hyperion from the Saturn.

Then, from this formula we can find the orbital period of the Hyperion:

$$P_H^2 = a_H^3 \frac{P_T^2}{a_T^3},$$

$$P_H = \sqrt{a_H^3 \frac{P_T^2}{a_T^3}} = \sqrt{(1.48 \cdot 10^9\ m)^3 \cdot \frac{(15.95\ days)^2}{(1.22 \cdot 10^9\ m)^3}} = 21.3\ days.$$

Answer:

$P_H = 21.3\ days.$