An object was launched with a velocity of 20 ms−1 at an angle of 45° to the vertical. At the top of its trajectory the object broke into two equal pieces. One piece fell vertically downwards. Where would the other piece fall? (Take g = 10 ms−2)

At the top of its trajectory the object has velocity:

$$v_y = 0$$

$$v_x = v_{x0} = v_0 \cos(\alpha)$$

Height of the object's trajectory:

$$H = \frac{v_{y0}^2}{2g} = \frac{(v_0 \sin(\alpha))^2}{2g}$$

Object will fall down (and go up) during time:

$$H = \frac{gt^2}{2} \to t = \sqrt{\frac{2H}{g}}$$

During this time, while object is go up, it will move at distance:

$$x_1 = v_{x0} t = v_0 \cos(\alpha) \sqrt{\frac{2H}{g}}$$

At the top of trajectory the object broke into two pieces. If the first piece fell vertically downwards, the second piece will move in x directions with velocity:

$$v_{x2} = 2v_x = 2v_0 \cos(\alpha)$$

While object is fall down, it will move at distance:

$$x_2 = v_{x2} t = 2v_0 \cos(\alpha) \sqrt{\frac{2H}{g}}$$

And total distance:

$$x = x_1 + x_2 = v_0 \cos(\alpha) \sqrt{\frac{2H}{g}} + 2v_0 \cos(\alpha) \sqrt{\frac{2H}{g}} = 3v_0 \cos(\alpha) \sqrt{\frac{2H}{g}} = 3v_0 \cos(\alpha) \sqrt{\frac{2\frac{(v_0 \sin(\alpha))^2}{2g}}{g}}$$

$$x = \frac{3v_0^2 \cos(\alpha) \sin(\alpha)}{g}$$

$$x = \frac{3 * (20m/s)^2 * \cos(45)\sin(45)}{10m/s^2} = 60m$$

Answer: $x = 60m$