Answer on Question #76338-Physics-Other

In a linear chain of two different types of atoms of masses m and M the displacement of the two atoms in the k=0 optical branch are u and v respectively. Show that u=−m vM

Solution

For a diatomic chain (of two different types of atoms of masses m and M that interact with same force constant C):

$$\frac{u}{v} = \frac{C(1 + e^{-ika})}{2C - M\omega^2}$$

In the k=0 optical branch:

$$\omega^2 = 2C\left(\frac{1}{m} + \frac{1}{M}\right)$$

Thus,

$$\frac{u}{v} = \frac{C(1 + e^{-i(0)a})}{2C - M\omega^2} = \frac{C(1+1)}{2C - M\omega^2} = \frac{C}{C - \frac{1}{2}M\omega^2}$$

$$\frac{u}{v} = \frac{C}{C - \frac{1}{2}M2C\left(\frac{1}{m} + \frac{1}{M}\right)} = \frac{1}{1 - \frac{M}{m} + 1} = -\frac{m}{M}.$$

$$u = -\frac{m}{M}v.$$