Answer on Question #76289-Physics-Mechanics-Relativity

Perfectly elastic oblique collision occurs between a ball A moving along the x-axis and a ball B at rest and of the same mass as ball A. After the collision, ball A moves at an angle of 30° with the X direction and ball B at an angle theta with the X axis. The value of theta is?

Solution

From the conservation of momentum:

X-direction:

$$mv = mv_a \cos 30 + mv_b \cos\theta$$

$$v = v_a \cos 30 + v_b \cos\theta$$

Y-direction:

$$0 = mv_a \sin 30 - mv_b \sin\theta$$

$$v_a \sin 30 = v_b \sin\theta$$

From the conservation of energy:

$$\frac{mv^2}{2} = \frac{mv_a^2}{2} + \frac{mv_b^2}{2}$$

$$v^2 = v_a^2 + v_b^2$$

Thus,

$$v = v_a \cos 30 + v_a \sin 30 \cot\theta = v_a(\cos 30 + \sin 30 \cot\theta)$$

$$v^2 = \left(\frac{v}{(\cos 30 + \sin 30 \cot\theta)}\right)^2 + \left(\frac{v \sin 30}{(\cos 30 + \sin 30 \cot\theta)\sin\theta}\right)^2$$

$$1 = \left(\frac{1}{(\cos 30 + \sin 30 \cot\theta)}\right)^2 + \left(\frac{\sin 30}{(\cos 30 + \sin 30 \cot\theta)\sin\theta}\right)^2$$

$$1 = \frac{1}{(\cos 30 + \sin 30 \cot\theta)^2}\left(1 + \frac{\sin^2 30}{\sin^2\theta}\right)$$

$$1 = \frac{1}{(\cos 30 \sin\theta + \sin 30 \cos\theta)^2}(\sin^2\theta + \sin^2 30)$$

$$1 = \frac{1}{(\sin(\theta + 30))^2}(\sin^2\theta + \sin^2 30)$$

Thus,

$$\theta = 60°$$

Answer: $60°$.