A solid sphere with a radius of 75 mm rolls (without slipping) down an inclined plane that is 5 m long. What is the angular velocity of the sphere at the bottom of the plane if it requires 10 seconds for it to reach the bottom? What is the angle of the plane?

Sphere will rotate at angle:

$$\varphi = \frac{S}{R}$$

From the other hand:

$$\varphi = \frac{\beta t^2}{2} \rightarrow \beta = \frac{2\varphi}{t^2} = \frac{2S}{Rt^2}$$

Angular velocity of the sphere at the bottom of the plane:

$$\omega = \beta t = \frac{2S}{Rt}$$

$$\omega = \frac{2 * 5m}{0.075m * 10s} = 13.33\ rad/s$$

Using the law of conservation of angular momentum:

$$I_A\beta = Mgl$$

Where I_A – moment of inertia of the sphere relative to the contact point, l is the length of the lever arm vector. Using Huygens–Steiner theorem:

$$\left(\frac{2}{3}MR^2 + MR^2\right)\beta = MgR \cdot sin(\alpha)$$

$$\frac{5}{3}R\beta = g \cdot sin(\alpha) \rightarrow sin(\alpha) = \frac{5R\beta}{3g}$$

$$\alpha = arcsin\left(\frac{5R\beta}{3g}\right) = arcsin\left(\frac{5R\frac{2S}{Rt^2}}{3g}\right) = arcsin\left(\frac{10S}{3gt^2}\right)$$

$$\alpha = arcsin\left(\frac{10 * 5m}{3 * 9.8m/s^2 * (10s)^2}\right) = 0.97°$$

Answer: $\omega = 13.33\ rad/s$, $\alpha = 0.97°$