Question #75613

Description:

Define thermodynamic probability (W) of the macrostate. Establish the relation between entropy (S) and W.

Solution.

Thermodynamic probability (W) the number of ways in which this macroscopic state of the system can be implemented,

such a value according to the probability theory has the following property, even if our system consists of two parts 1 and 2 then we will have

$$W = W_{12} = W_1 W_2, \;\; S_{12} = S_1 + S_2$$

since entropy is defined as a measure of disorder in a system of bodies, it clearly depends on the probability of a given state and hence on W, we differentiate the function S=f(W) by W_1 and W_2

$$W_{12} = W_1 W_2, \;\; S_{12}(W_{12}) = S_{12}(W_1 W_2) = S_1(W_1) + S_2(W_2),$$

$$\frac{\partial S_{12}}{\partial W_1} = W_2 * \; S'(W_1 W_2) = S'(W_1), =>$$

$$\frac{\partial^2 S_{12}}{\partial W_1 \, \partial W_2} = \; S'(W_1 W_2) + W_1 W_2 \, S''(W_1 W_2) = 0$$

$$S'(W) + W \, S''(W) = 0$$

the solution of this differential equation has the form

$$S = k_B \ln W$$

where k_B is called Boltzmann constant =1.38 10^{-23}J/K

for gas with particles N

$$W = \frac{N!}{N_1! * N_2! * N_3! * \dots}$$

where N is the total number of molecules of the gas in the considered volume. Ni-number of molecules, moving at speeds corresponding to the i-th cell of the conditional velocity space

Answer

$$S = k_B \ln W$$

for gas with particles N

$$W = \frac{N!}{N_1! * N_2! * N_3! * \dots}$$