

Obtain an expression for the time period of a satellite orbiting the earth. At what altitude should a satellite be placed for its orbit to be geosynchronous?

Second Newton's law for this case:

$$G \frac{M_E m}{(R_E + h)^2} = m \frac{v^2}{R_E + h}$$

$$G \frac{M_E}{R_E + h} = v^2$$

$$v = \sqrt{G \frac{M_E}{R_E + h}} = \sqrt{\frac{g R_E^2}{R_E + h}} = R_E \sqrt{\frac{g}{R_E + h}}$$

One revolution will be made during time (period):

$$T = \frac{2\pi(R_E + h)}{R_E \sqrt{\frac{g}{R_E + h}}} = \frac{2\pi(R_E + h)^{3/2}}{R_E \sqrt{g}}$$

Assuming that the Earth period is 24h:

$$\frac{2\pi(R_E + h)^{3/2}}{R_E \sqrt{g}} = 24 * 3600s$$

$$h = \left(\frac{86400s * R_E \sqrt{g}}{2\pi} \right)^{2/3} - R_E$$

$$h = \left(\frac{86400s * 6400000m * \sqrt{9.8m/s^2}}{2 * 3.14} \right)^{2/3} - 6400000m = 3.6 * 10^7m = 3.6 * 10^4km$$

Answer: $T = \frac{2\pi(R_E+h)^{3/2}}{R_E\sqrt{g}}$, $h = 3.6 * 10^4km$