Answer on Question 74501, Physics, Other

Question:

Obtain the rotational kinetic energy of the Earth due its daily rotation on its axis. Assume the Earth to be a uniform sphere; take $m = 5.98 \cdot 10^{24}\ kg$; $r = 6.37 \cdot 10^{6}\ m$.

Solution:

We can find the rotational kinetic energy of the Earth from the formula:

$$KE_{rot} = \frac{1}{2} I \omega^2,$$

here, I is the moment of inertia of the Earth (which we assume to be a uniform sphere), ω is the angular velocity of the sphere.

We can find the moment of inertia of the uniform sphere from the formula:

$$I = \frac{2}{5} m r^2,$$

here, m is the mass of the sphere, r is the radius of the sphere.

We can find the angular velocity of the sphere from the formula:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{1\ day} = \frac{2\pi}{24 \cdot 3600\ s} = \frac{2\pi}{86400\ s},$$

here, $\Delta\theta = 2\pi$ is the change of the angular displacement of the sphere relative to the origin, $\Delta t = 1\ day = 86400\ s$ is the change in time in which that displacement took place.

Finally, substituting I and ω into the formula for the rotational kinetic energy of the Earth, we get:

$$KE_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \omega^2 =$$

$$= \frac{1}{5} \cdot 5.98 \cdot 10^{24}\ kg \cdot (6.37 \cdot 10^{6}\ m)^2 \cdot \left(\frac{2\pi}{86400\ s}\right)^2 = 2.56 \cdot 10^{29}\ J.$$

Answer:

$KE_{rot} = 2.56 \cdot 10^{29}\ J.$