Answer on Question #74431, Physics / Other

Two waves, travelling along the same direction, are given by

y1(x, t) = asin (w1t − k1x) and y2 (x, t) = asin (w2t − k2x)

Suppose that the values of w1 and k1 are respectively slightly greater than w2 and k2.

i) Obtain an expression for the resultant wave due to their superposition.

ii) Explain the formation of wave packet.

Solution:

(i) Let the waves are

$$y_1 = a\sin(\omega_1 t - k_1 x)$$
$$y_2 = a\sin(\omega_2 t - k_2 x)$$

The superposition of these two waves is given by

$$y = y_1 + y_2 = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

Using the trigonometric relation

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

we write the above equation as

$$y = y_1 + y_2 = 2a\sin\left[\frac{(\omega_1+\omega_2)}{2}t - \frac{(k_1+k_2)}{2}x\right]\cos\left[\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right] =$$
$$= 2a\sin[\omega t - kx]\cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right]$$

where $\omega = (\omega_1+\omega_2)/2$, $k = (k_1+k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$.

(ii) The resultant equation shows beats.

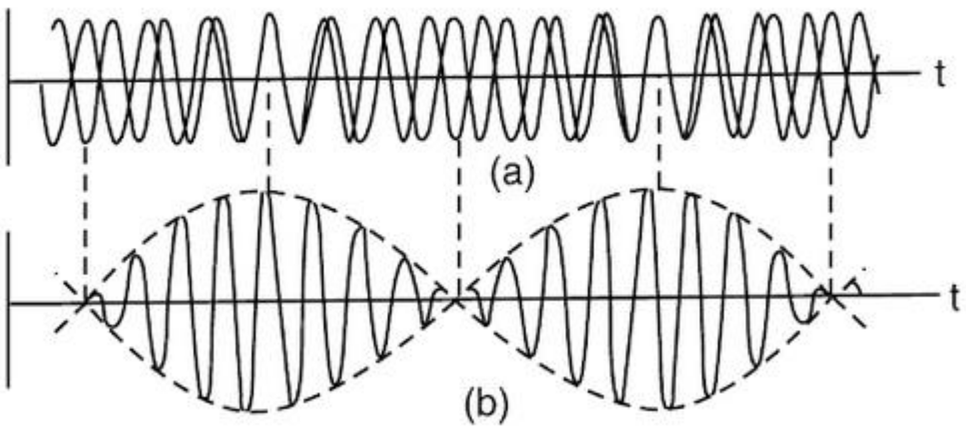

Fig. Beats are formed when two waves of slightly different frequencies combine

(a) The individual waves; (b) the resultant wave.

The resultant wave is seen to have the following two parts:

- A wave of angular frequency ω and propagation constant k, moving with a velocity

$$v_p = \frac{\omega}{k} = \nu\lambda$$

- A second wave of angular frequency $\frac{\Delta\omega}{2}$ and propagation constant $\Delta k/2$, moving with a velocity

$$v_g = \frac{\Delta\omega}{\Delta k}$$

When a number of plane waves of slightly different wavelengths travel in the same direction, they form wave groups or wave packets. The velocity with which the wave group advances in the medium is known as the group velocity v_g. Each component wave has its own phase velocity, $v_p = \nu\lambda$. The wave packet has amplitude that is large in a small region and very small outside it. Such a variation of amplitude is called the modulation of the wave. The velocity of propagation of the modulation is known as the group velocity, v_g.

Answer: (i) $y = 2a\sin[\omega t - kx]\cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right]$ where $\omega = (\omega_1 + \omega_2)/2$, $k = (k_1 + k_2)/2$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$.