Answer on Question 74125, Physics, Astronomy, Astrophysics

Question:

Calculate the value of acceleration due to gravity at point:

a) 5.0 km above the Earth`s surface and

b) 5.0 km below the Earth`s surface.

Radius of Earth is $6400\,km$ and the value of g at the surface of the Earth is $9.8\ m/s^2$.

Solution:

a) As we know, the acceleration due to gravity on the surface of the Earth is given by the formula:

$$g = \frac{GM}{R^2}, (1)$$

here, G is the universal gravitational constant, M is the mass of the Earth, R is the radius of the Earth.

At the height h above the Earth, the acceleration due to gravity is given by:

$$g_h = \frac{GM}{(R+h)^2}. (2)$$

Let's divide equation (2) by equation (1):

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \cdot \frac{R^2}{GM} = \frac{R^2}{(R+h)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2} = \left(1-\frac{2h}{R}\right).$$

Finally, we get:

$$g_h = g\left(1-\frac{2h}{R}\right) = 9.8\ \frac{m}{s^2} \cdot \left(1-\frac{2 \cdot 5 \cdot 10^3\ m}{6.4 \cdot 10^6\ m}\right) = 9.78\ \frac{m}{s^2}.$$

b) Let's denote the density of the Earth as ρ. Then, the mass of the Earth can be written as follows:

$$M = \rho V = \frac{4}{3}\pi R^3 \rho,$$

here, $V = \frac{4}{3}\pi R^3$ is the volume of the Earth.

Then, we can write the acceleration due to gravity at the surface of the Earth:

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \cdot \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi G R \rho \ (3)$$

At the depth d below the Earth's surface the acceleration due to gravity is given by:

$$g_d = \frac{4}{3}\pi G (R - d)\rho \ (4)$$

Let's divide equation (4) by equation (3):

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G (R-d)\rho}{\frac{4}{3}\pi G R \rho} = \frac{R-d}{R} = \left(1 - \frac{d}{R}\right).$$

Finally, we get:

$$g_d = g\left(1 - \frac{d}{R}\right) = 9.8\ \frac{m}{s^2} \cdot \left(1 - \frac{5 \cdot 10^3\ m}{6.4 \cdot 10^6\ m}\right) = 9.79\ \frac{m}{s^2}.$$

Answer:

a) $g_h = 9.78\ \frac{m}{s^2}$.

b) $g_d = 9.79\ \frac{m}{s^2}$.