Answer on Question #73944, Physics / Other

the energy of a particle of mass m bound by a special kind of spring is E=p^2/2m +kx^4 where k is a positive constant .use the heisenbergs uncertainty principle to calculate the minimum possible energy of the particle.

Solution:

From uncertainty principle let

$$p\,x \approx \frac{\hbar}{2}$$

So,

$$p = \frac{\hbar}{2x}$$

Then

$$E = \frac{\hbar^2}{8mx^2} + \frac{kx^2}{2}$$

Then we differentiate to find location of minimum *E*

$$\frac{dE}{dx} = -\frac{2\hbar^2}{8mx^3} + kx = 0$$

$$x = \left[\frac{\hbar^2}{4mk}\right]^{\frac{1}{4}}$$

Substituting this into the energy equation to find minimum energy E_0

$$E_0 = \frac{\hbar^2}{8m\sqrt{\frac{\hbar^2}{4mk}}} + \frac{k\sqrt{\frac{\hbar^2}{4mk}}}{2} = \left(\frac{\hbar}{4}\right)\sqrt{\frac{k}{m}} + \left(\frac{\hbar}{4}\right)\sqrt{\frac{k}{m}} = \frac{\hbar}{2}\sqrt{\frac{k}{m}}$$

Answer: $\frac{\hbar}{2}\sqrt{\frac{k}{m}}$