Answer on Question #73918, Physics / Mechanics | Relativity

A compressed spring is used to propel a ball bearing along a track which contains a circular loop of radius 0.10m in a vertical plane. The spring obeys Hooke's law and requires a force of 0.20N to compress it by 1.0mm.

(a) The spring is compressed by 30 mm. Calculate the energy stored in the spring.

(b) A ball bearing of mass 0.025kg is placed against the end of the spring which is then released. Calculate,

(I) the speed with which the ball bearing least the spring

(ii) the speed of the ball at the top of the loop

(iii) the force exerted on the ball by the track at the top of the loop

Assume that the effects of friction can be ignored.

Solution:

a)

$$F = kx$$

$$k = \frac{F}{x}$$

$$k = \frac{0.2\ N}{0.001\ m} = 200\ N/m$$

We use the potential energy equation of a linear spring.

$$U = \frac{kx^2}{2}$$

$$U = \frac{200\frac{N}{m} \times (0.03\ m)^2}{2} = 0.09J$$

b)(i)

$$KE\ = \ U$$

$$\frac{mv^2}{2} = U$$

$$v = \sqrt{\frac{2U}{m}}$$

$$v = \sqrt{\frac{2 \times 0.09J}{0.025kg}} = 2.7\ ms^{-1}$$

b)(ii)

$$KE_b + PE_b = \ KE_t + PE_t$$

$$\frac{mv_b^2}{2} + \mathrm{mg}h_b = \frac{mv_t^2}{2} + \mathrm{mg}h_t$$

$$h_b = 0$$

We get

$$\frac{v_b^2}{2} = \frac{v_t^2}{2} + \mathrm{g}h_t$$

$$v_b^2 = v_t^2 + 2\mathrm{g}h_t$$

$$v_t = \sqrt{v_b^2 - 2gh_t}$$

Where $h_t = 2R$

$$v_t = \sqrt{(2.7\ m/s)^2 - 2 \times 9.8\ \frac{m}{s^2} \times 0.2\ m} = 1.8\ m/s$$

b)(iii)

$$\sum F = N + mg$$

$$F = m\frac{v^2}{R}$$

$$m\frac{v^2}{2} = N + mg$$

$$N = m\left(\frac{v^2}{R} - g\right)$$

$$N = 0.025kg \times \left(\frac{(1.8\ m/s)^2}{0.1m} - 9.8\frac{m}{s^2}\right) = 0.56\ N$$

Answer: a) 0.09 J; b) (i) 2.7 ms^{-1}, (ii) 1.8 ms^{-1}, (iii) 0.56 N