Answer on Question # 73742, Physics -Electric Circuits:

Question: The magnitude of work done in taking a unit positive charge in electric field E from point A to point B is given by:

$$W = \int -E.dr$$

Show that the value of the line integral of the electric field (right hand side of the above equation) does not depend on the path taken to move the unit positive charge from point A to B.

Solution: We know for conservative force field $\nabla \times E = 0$ i.e; $E = -\nabla V$ (where V is the potential)

Now, work done in taking a unit positive charge in electric field E from point A to point B is given by: $W = \int -E.dr$ .....(1)

Put $E = -\nabla V$ in equation (1) and we get,

$$W = \int -(-\nabla V).dr = \int dV \quad .....(2) \qquad [As\ ,\ ((\nabla V).dr) = dV]$$

Now, integrating right hand side of equation (2) from point A to B , we get,

$$W = V_B - V_A \quad .....(3)$$

So, equation (3) depends on initial point (A) and final point (B) only.

Answer : So, the value of the line integral of the electric field does not depend on the path taken to move the unit positive charge from point A to B only depends on the initial and final point.