Answer on Question #73582, Physics / Other

show that an arbitrary reciprocal lattice vector g=ha1+ka2+a3 is perpendicular to the family of planes denoted by in the direct lattice space

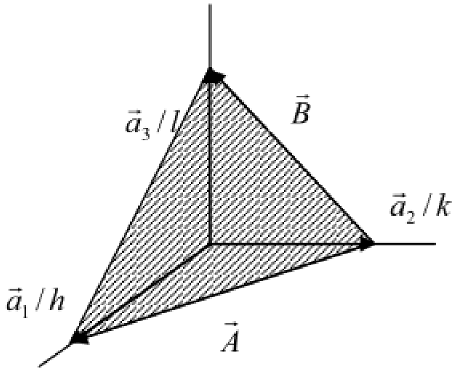
Solution:

Consider a plane hkl in a crystal lattice.

We will prove that the reciprocal lattice vector $\boldsymbol{G} = h\boldsymbol{b_1} + k\boldsymbol{b_2} + l\boldsymbol{b_3}$ is perpendicular to this plane.

To prove that the reciprocal lattice vector $\boldsymbol{G} = h\boldsymbol{b_1} + k\boldsymbol{b_2} + l\boldsymbol{b_3}$ is perpendicular to this plane, it suffices to show that $\boldsymbol{G}$ is perpendicular to two nonparallel vectors in this plane.

For the plane (hkl), it intercepts axis $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, and $\boldsymbol{a_3}$ at a ratio $\frac{1}{h}:\frac{1}{k}:\frac{1}{l}$.



The two vectors in the plane can be chosen as $\boldsymbol{A} = \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{k}\boldsymbol{a_2}\right)$ and $\boldsymbol{B} = \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{l}\boldsymbol{a_3}\right)$.

Obviously, they are not parallel to each other.

Recall that $\boldsymbol{b} \cdot \boldsymbol{a} = 2\pi\delta_{ij}$, we have from direct calculation:

$$\boldsymbol{G} \cdot \boldsymbol{A} = \boldsymbol{G} \cdot \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{k}\boldsymbol{a_2}\right) = (h\boldsymbol{b_1} + k\boldsymbol{b_2} + l\boldsymbol{b_3}) \cdot \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{k}\boldsymbol{a_2}\right) =$$

$$= h\boldsymbol{b_1}\frac{\boldsymbol{a_1}}{h} - k\boldsymbol{b_2}\frac{\boldsymbol{a_2}}{k} = 2\pi - 2\pi = 0$$

$$\boldsymbol{G} \cdot \boldsymbol{B} = \boldsymbol{G} \cdot \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{l}\boldsymbol{a_3}\right) = (h\boldsymbol{b_1} + k\boldsymbol{b_2} + l\boldsymbol{b_3}) \cdot \left(\frac{1}{h}\boldsymbol{a_1} - \frac{1}{l}\boldsymbol{a_3}\right) =$$

$$= l\boldsymbol{b_3}\frac{\boldsymbol{a_3}}{l} - k\boldsymbol{b_2}\frac{\boldsymbol{a_2}}{k} = 2\pi - 2\pi = 0$$

Therefore $\boldsymbol{G} \perp \boldsymbol{A}$ and $\boldsymbol{G} \perp B$ so $\boldsymbol{G}$ is perpendicular to the (hkl) plane.

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