

Answer on question #73505-physics-mechanics|relativity.

For a damped oscillator, amplitude will decrease with time because of resistive forces. This has the effect of energy decreasing with time too.

The equation for the displacement of a forced oscillator is given as

$$\frac{d^2x}{dt^2} + \omega^2 x + \frac{dx}{dt} 2b = f \cos \omega t$$

The general solution to this differential equation is given as $x(t) = x_1(t) + x_2(t)$ where $x_1(t)$ is the complementary function and $x_2(t)$ is the particular integral. In transient state, $x_1(t)$ decays exponentially to zero as time increases hence the damped oscillation of the system dies off. The system now oscillates with the driving force frequency called the steady state. The solution of the equation at steady state is given as

$$x_2(t) = A \cos(\omega t - \delta) = f \cos(\omega t)$$

Velocity of the system, $\frac{dx_2(t)}{dt} = a \omega \sin(\omega t - \delta)$.

Average power absorbed by the system

Power = force \times velocity, $f(t) = f \cos(\omega t)$ and $v = a \omega \sin(\omega t - \delta)$.

$$\langle p \rangle = \omega f a \cos \omega t (\sin \omega t \cos \delta - \cos \omega t \sin \delta), = \omega f A \cos \omega t \sin \omega t \cos \delta - \omega f A \cos^2 \omega t \sin \delta$$

Taking one complete cycle of power over time average, $\langle \cos \omega t \sin \omega t \rangle = 0$ and $\langle \cos^2 \omega t \rangle = 1/2$

$$\langle p \rangle = \omega f 1/2 \sin \delta, \text{ but } \sin \delta = \frac{2b\omega}{\sqrt{(\omega^2 - \omega^2)^2 + 4b^2\omega^2}}$$

$$\langle p \rangle = \omega f 1/2 \times \frac{2b\omega}{\sqrt{(\omega^2 - \omega^2)^2 + 4b^2\omega^2}}$$

$$\langle p \rangle = \frac{b F^2}{m} \times \frac{\omega^2}{(\omega^2 - \omega^2)^2 + 4b^2\omega^2}$$

Reference

1. Dr. M ghosh, Dr. D Bhattacharyya (2014) oscillations, waves and acoustics.