Answer on Question #73476 - Physics / Mechanics | Relativity

A harmonic wave on a rope is described by

$$y(x,t) = (4.00\text{ mm})\sin\left(\frac{2\pi}{0.82\text{m}}\left(\left(10\frac{\text{m}}{\text{s}}\right)t + x\right)\right)$$

i) Calculate the wavelength and time period of the wave.

ii) Determine the displacement and acceleration of the element of the rope located at $x = 0.58$ m at time, $t = 0.41$ s.

Solution:

The general form of the wave equation

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(vt + x)\right)$$

i) Where λ is a wavelength, v is a wave velocity.

Thus

Wavelength $\lambda = 0.82$ m

Time period $T = \frac{\lambda}{v} = \frac{0.82}{10} = 0.082$ s

ii) The displacement at the $x = 0.58$ m and $t = 0.41$ s

$$y(x,t) = 4.00\sin\left(\frac{2\pi}{0.82}(10 \times 0.41 + 0.58)\right) = -3.857\text{ mm}$$

The acceleration

$$a = y'' = -0.004 \times \left(\frac{2\pi}{0.82} \times 10\right)^2 \sin\left(\frac{2\pi}{0.82}(10 \times 0.41 + 0.58)\right) = 22.65\ \frac{\text{m}}{\text{s}^2}$$

Answers:

Wavelength $\lambda = 0.82$ m

Time period $T = \frac{\lambda}{v} = \frac{0.82}{10} = 0.082$ s

The displacement $y(x,t) = -3.857$ mm

The acceleration $a = 22.65\ \frac{\text{m}}{\text{s}^2}$